SPECIALIST MATHEMATICS WORKED SOLUTIONS
ERRATA (as at 2 October 2008)

EXERCISE 1E.2 (page 26)

2 a The first part of the solution should be:

$$\sin^2 x + \sin x - 2 = 0$$
$$\therefore \quad (\sin x + 2)(\sin x - 1) = 0$$

c The last line of the solution should be:

$$\therefore \quad x = \frac{2\pi}{3} + k2\pi \quad \text{or} \quad x = \frac{4\pi}{3} + k2\pi \quad \text{or} \quad x = 0 + 2k\pi$$

EXERCISE 2D.1 (page 49)

4 d The last part of the solution should be:

However if $a = 0$, $a + ai = 0$ (i.e., is *not* complex) so the other root could be any real number.

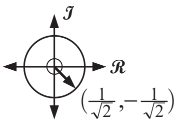
But if $a = 0$ is a root, then $x = 0$ is a solution to the equation.

$\therefore \quad b = 0$

$\therefore \quad a = 0, \quad b = 0$ is also a solution.

EXERCISE 2D.1 (page 49)

4 d Diagram should be:



EXERCISE 2G.2 (page 65)

1 a iii The last line of the solution should be:

$$\therefore \quad z = \frac{1 - w^n}{2} \quad \text{where} \quad n = 0, 1, 2 \quad \text{and} \quad w = \text{cis}\left(\tfrac{2\pi}{3}\right)$$

EXERCISE 3G.2 (page 104)

3 $2 - 3i$ is a zero of $z^3 + pz + q$ and as the cubic has real coefficients, $2 + 3i$ is also a zero

For $2 \pm 3i$, $\alpha + \beta = 4$ and $\alpha\beta = 4 - 9i^2 = 13$

$\therefore \quad x^2 - 4x + 13$ is a factor

$\therefore \quad z^3 + pz + q = (x^2 - 4x + 13)(x + a)$ for some a.

	1	-4	13
$\times$		1	a
	a	$-4a$	$13a$
1	-4	13	
1	$a-4$	$13-4a$	$13a$

Equating coefficients:

$a - 4 = 0, \quad 13 - 4a = p \quad$ and $\quad 13a = q$

$\therefore \quad a = 4, \quad p = -3, \quad q = 52$

$\therefore$ the other zeros are -4 and $2 + 3i$

Check:

Since $P(2 - 3i) = 0$, $(2 - 3i)^3 + p(2 - 3i) + q = 0$

Expanding, $(-46 - 9i) + p(2 - 3i) + q = 0$

$\therefore \quad (-46 + 2p + q) + (-9 - 3p)i = 0$

Equating real and imaginary parts,

$-46 + 2p + q = 0 \quad$ .....(1)

and $\quad -9 - 3p = 0 \quad$ .....(2)

From (2), $p = -3$, so in (1), $-46 - 6 + q = 0$

$\therefore \quad p = -3, \quad q = 52$

EXERCISE 3J.2 (page 110)

3 a For $z \to z^2$

$\therefore \quad z_0 = r \text{ cis } \theta$
$z_1 = r^2 \text{ cis } 2\theta$
$z_2 = r^4 \text{ cis } 4\theta$
$\vdots$
$z_{n-1} = r^{2^{n-1}} \text{ cis } 2^{n-1}\theta$
$z_n = r^{2^n} \text{ cis } 2^n\theta$

The series diverges if the magnitude is increasing

i.e., $r^{2^n} > r^{2^{n-1}} \quad \{\text{since } r \geqslant 0\}$

$\therefore \quad r^{2^n - 2^{n-1}} > 1$

$\therefore \quad r^{2^{n-1}(2-1)} > 1$

$\therefore \quad r^{2^{n-1}} > 1$

$\therefore \quad r > 1^{\frac{1}{2^n - 1}}$

$\therefore \quad r > 1$

b The series converges if the magnitude is decreasing:

$\therefore \quad r^{2^n} < r^{2^{n-1}} \quad \{\text{since } r \geqslant 0\}$

$\therefore \quad r^{2^n - 2^{n-1}} < 1$

$\therefore \quad r^{2^{n-1}(2-1)} < 1$

$\therefore \quad r^{2^{n-1}} < 1$

$\therefore \quad r < 1^{\frac{1}{2^n - 1}}$

$\therefore \quad r < 1$

$\therefore$ since $r \geqslant 0$,

$0 \leqslant r < 1$

EXERCISE 4I (page 159)

7 b Answer should read: The line meets the YOZ plane...

10, 11 Diagrams should be labelled with vector **v**, not **u**.

EXERCISE 4J (page 170)

14 b First line of working should be: $\therefore$ using **a**, $d = \frac{|\overrightarrow{QP} \bullet \mathbf{n}|}{|n|}$

EXERCISE 6A.1 (page 221)

1 c these lines of working are missing:

$\therefore \quad v'(t) = 5.2418 \times 1.0865 \times t^{0.0865}$

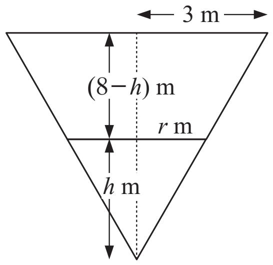
$\doteqdot 5.6952 \times t^{0.0865}$

The table should be:

Time (s)	*Estimate*	*Model*
0	0.00	0.00
4	6.25	6.42
8	6.75	6.82
12	7.00	7.06
16	7.25	7.24
20	7.50	7.38
24	7.50	7.50
28	7.50	7.60
32	7.50	7.69

REVIEW SET 6B (page 249)

6



a $V(r) = \frac{1}{3}\pi r^2 h$

Using similar triangles $\frac{h}{r} = \frac{8}{3}$

$\therefore \quad h = \frac{8r}{3}$

$\therefore \quad V(r) = \frac{1}{3}\pi r^2 \left(\frac{8r}{3}\right) = \frac{8\pi}{9} r^3$

b *Particular case:* When $h = 5$, $r = \frac{3h}{8} = \frac{15}{8}$ and $\frac{dV}{dt} = -0.2 = -\frac{1}{5}$

Now $\frac{dV}{dt} = \frac{8\pi r^2}{3}\frac{dr}{dt}$

$\therefore \quad -\frac{1}{5} = \frac{8\pi(\frac{15}{8})^2}{3}\frac{dr}{dt}$

$\therefore \quad -\frac{1}{5} = \frac{225}{24}\pi\frac{dr}{dt}$

$\therefore \quad \frac{dr}{dt} = -\frac{8}{375\pi}$

$\therefore \quad \frac{dr}{dt} \doteqdot -0.00679$

i.e., the radius is decreasing at 0.00679 m/minute

REVIEW SET 6C (page 249)

1 **a** X23 has direction vector $[x_1, y_1] = [2, 4] + t[1, -3]$

i.e., $x_1 = 2 + t$, $y_1 = 4 - 3t$, $t \geqslant 0$

b Y18 has direction vector

$[x_2, y_2] = [11, 3] + (t - 2)[-1, a]$

i.e., $x_2 = 11 - (t - 2)$, $y_2 = 3 + a(t - 2)$, $t \geqslant 2$

c Interception occurred when $x_1 = x_2$ and $y_1 = y_2$ for the same t value,

i.e., $2 + t = 11 - (t - 2)$

$\therefore \quad 2t = 11$

$\therefore \quad t = 5.5$

and if $t = 5.5$, and $y_1 = y_2$

then $4 - 16.5 = 3 + 3.5a$

$\therefore \quad 3.5a = -15.5$

$\therefore \quad a \doteqdot -\frac{31}{7} \doteqdot -4.429$

$\therefore$ interception occurred 5.5 minutes after 2:17
i.e., at 2:22:30 pm.

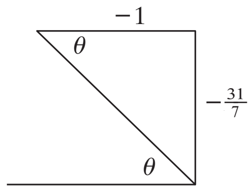
d The intercepting torpedo was Y18, where $x_2 = 13 - t$, $y_2 = 3 + a(t - 2)$

where $a = -\frac{31}{7}$ in order for interception to occur.

$\therefore \quad \mathbf{v}_{Y18} = [-1, -\frac{31}{7}]$

$\therefore \quad \text{speed} = \sqrt{(-1)^2 + (-\frac{31}{7})^2}$

$\doteqdot 4.540$ km/min



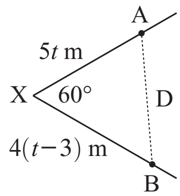
$\tan\theta = \frac{-\frac{31}{7}}{-1} = \frac{31}{7}$ and $\theta \doteqdot 77.28^o$

6 **Note**: the question should have stated that B passes through X 3 *seconds* after A passes through X.

Let t be the number of seconds after A passes through X. In this time, A travels $5t$ m.

Now B passes through X when $t = 3$.

$\therefore$ for $t > 3$, B is $4(t - 3)$ m from X.



Using the cosine rule,

$$D^2 = 25t^2 + 16(t-3)^2 - 2 \times 5t \times 4(t-3) \times \cos 60^o$$
$$= 25t^2 + 16(t-3)^2 - 20t(t-3)$$

$\therefore \quad 2D\frac{dD}{dt} = 50t + 32(t-3) - 20(t-3) - 20t$

When $5t = 20$, $t = 4$

and $D^2 = 25 \times 16 + 16 - 20 \times 4$
$= 400 + 16 - 80$
$= 336$

$\therefore \quad 2\sqrt{336}\,\frac{dD}{dt} = 200 + 32 - 20 - 20 \times 4$

$\therefore \quad 2\sqrt{336}\,\frac{dD}{dt} = 132$

$\therefore \quad \frac{dD}{dt} = \frac{66}{\sqrt{336}} \doteqdot 3.60$ m/s

and so the distance increases at 3.60 m/s

EXERCISE 7C (page 270)

8 Second and third lines of working should be:

Then $\frac{d\phi}{dt} = -100$ revolutions per second {clockwise
$= -200\pi$ radians per second rotation}

a Last two lines of working should be:

$= \frac{1}{2} \times \frac{-\frac{1}{2}}{\frac{\sqrt{13}}{4}} \times (-200\pi)$ {as $\frac{d\phi}{dt} = -200\pi$ ^{c}/sec}

$= \frac{200\pi}{\sqrt{13}}$, i.e., θ is increasing at $\frac{200\pi}{\sqrt{13}}$ radians per second

b Last two lines of working should be:

$= \frac{1}{2} \times \frac{-1}{1} \times (-200\pi)$

$= 100\pi$, i.e., θ is increasing at 100π radians per second

EXERCISE 7D.1 (page 272)

1 e

$$\begin{aligned} \cot\theta &= (\tan\theta)^{-1} \\ \therefore\quad \frac{d}{dx}(\cot\theta) &= -(\tan\theta)^{-2}(\sec^2\theta) \\ &= -\left(\frac{\cos\theta}{\sin\theta}\right)^2\left(\frac{1}{\cos\theta}\right)^2 \\ &= -\frac{1}{\sin^2 x} \\ &= -\text{cosec}^2\,\theta \end{aligned}$$

$\therefore\quad \int \text{cosec}^2\,\theta\,dx = -\cot x + c$

EXERCISE 8A.1 (page 296)

1 a

$$\begin{aligned} P(t) &= P_0 2^{\frac{t}{30}} \\ \text{now}\quad t &= 2\text{ hours} = 120\text{ mins} \\ \text{and}\quad P(120) &= 1.6\times 10^7 \\ \therefore\quad P_0 2^{\frac{120}{30}} &= 1.6\times 10^7 \\ \therefore\quad P_0 &= \frac{1.6\times 10^7}{2^4} \\ &= 10^6 \end{aligned}$$

$\therefore$ the initial population was $= 10^6$ bacteria

b i $P(30) = 2^{\frac{30}{30}}P_0 = 2P_0 = 2\times 10^6$

ii $P(60) = 2^2P_0 = 4P_0 = 4\times 10^6$

iii $t = 3$ hours $= 180$ mins

$P(180) = 2^6P_0 = 64P_0 = 6.4\times 10^7$

EXERCISE 8D.2 (page 309)

1 a Delete last three lines of working and replace with:
$\therefore\quad y = Ax^3$

b Replace last line of working with: $\therefore\quad y = Ae^{\frac{x^2}{8}}$

c Replace last line of working with: $\therefore\quad y = Ae^{e^x}$

EXERCISE 9A.3 (page 343)

5 Solution should finish:

$\therefore$ since $y(0) = V(0) = -2I_p < 0,\ \phi = \frac{3\pi}{2}$
Now $B\sin\phi = -2I_p$
$\therefore -B = -2I_p \quad \therefore\quad B = 2I_p$ and $A = \frac{1}{2}B = I_p$
$\therefore\quad I(t) = I_p\cos\left(t+\frac{3\pi}{2}\right)$ and $V(t) = 2I_p\sin\left(t+\frac{2\pi}{3}\right)$