HAESE MATHEMATICS
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# TEACHER NOTES FOR YEAR 11 MATHEMATICAL METHODS

## CHAPTER 0: BACKGROUND KNOWLEDGE (ONLINE)

| | | SACE | ACARA |
|---|---|---|---|
| A | Coordinate geometry | Topic 1 | Unit 1 |
| B | The equation of a line | Sub-topic 1.1 | Topic 1 |
| C | The intersection of lines | | |

We put this material in a separate online chapter because the students have seen it several times and should know it thoroughly.

If classes require this material, they need to move through it swiftly. Otherwise, they will struggle to get through the course.

We have left out "Solve linear equations" completely, because a student unable to do this in Year 11 should not be attempting this course.

## CHAPTER 1: RELATIONSHIPS BETWEEN VARIABLES

| | | SACE | ACARA |
|---|---|---|---|
| A | Linear relationships | Topic 1<br>Sub-topic 1.1 | Unit 1<br>Topic 1 |
| B | Inverse proportion | Topic 1 | Unit 1 |
| C | Reciprocal functions | Sub-topic 1.2 | Topic 1 |
| D | Rational functions | | |

In this chapter we provide brief introductions into direct and inverse proportionality. It is important to not linger on this chapter, since proportionality is not returned to in the course. The rational functions are written to reflect the syllabus. However, it seems strange to not consider a general translation of the hyperbola $y = \frac{1}{x}$.

It is important that students understand the difference between inverse proportionality and a general rational function. Just as $y \propto x$ or $y = mx$ is the special case of $y = mx + c$, so is $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$ the special case of $y = \frac{a}{x-b}$.

## CHAPTER 2: QUADRATICS

| | | SACE | ACARA |
|---|---|---|---|
| A | Solving quadratic equations | | |
| B | The discriminant of a quadratic | | |
| C | The sum and product of the roots | | |
| D | Quadratic functions | Topic 2 | Unit 1 |
| E | Finding a quadratic from its graph | Sub-topic 2.1 | Topic 1 |
| F | Where functions meet | | |
| G | Problem solving with quadratics | | |
| H | Quadratic optimisation | | |

When considering the order for presenting material in this book, we are mindful of how things will actually work in the classroom. Usually, the first few weeks of term are difficult because it takes students time to settle down into a regular work pattern.

Knowing that Chapter 1 presents some new material, we want to help students settle by the revision and extension of quadratics from previous years *before* we get to the rigours of formal function notation, domain and range, and function notation. We are content to use the word "function", however, since students had an introduction to functions in Year 10. The other purpose is that when we introduce domain and range in the functions chapter, we can immediately use quadratics in addition to linear functions and graphs.

## CHAPTER 3: RELATIONS AND FUNCTIONS

| | | SACE | ACARA |
|---|---|---|---|
| A | Relations and functions | | |
| B | Function notation | | |
| C | Domain and range | Topic 1 | Unit 1 |
| D | Composite functions | Sub-topic 1.3 | Topic 1 |
| E | Sign diagrams | | |
| F | Transforming $y = f(x)$ | | |
| G | Graphs of circles | | |

Having studied quadratics in detail, we now consider relations and functions more formally. The chapter is somewhat fragmented, which tends to happen in any introduction to functions. This highlights the case for dealing with quadratics first, enabling students to settle in to work. Having done domain and range, and composite functions, we move to sign diagrams which will have later virtue in polynomials and calculus. We highlight $y^2 = x$ as an example of a relation which is not a function, since it has been given specific mention in the syllabus.

The transformations covered in Section F have been removed from the SACE syllabus. However, we would encourage South Australian classes to study this Section, as it provides a valuable background for studying trigonometric and exponential functions later in the year.

When considering the transformations listed in the ACARA syllabus, reflections are not mentioned. There is clearly an assumption, however, that the parameters $c$ and $k$ in $y = cf(x)$ and $y = f(kx)$ can be negative, amounting as it were to a dilation and reflection combined. We feel this is unnatural and foolish for many reasons, including:

- it leads to non-unique specification of transformations, which is mathematically incorrect
- it leads to other errors such as defining polar coordinates in which $r$ can be negative
- it leads away from the Specialist Mathematics work on linear transformations using matrices, in which reflections *are* given a specific mention.

Therefore, we are including reflections in their own right, and considering the combination of transformations of functions as an example of the use of composition of functions.

We close the chapter with circles because they provide a further example of the use of transformations, especially translations to locate the centre. They also belong under the banner of relations, which are then largely put aside for the remainder of the course. Having studied quadratics already, we reinforce the method of "completing the square" by applying it to circles.

## CHAPTER 4: POLYNOMIALS

| | | SACE | ACARA |
|---|---|---|---|
| A | Polynomials | | |
| B | Zeros, roots, and factors | | |
| C | Factorising cubic polynomials | Topic 2 | Unit 1 |
| D | Solving cubic equations | Sub-topic 2.2 | Topic 1 |
| E | Graphing cubic functions | | |
| F | Quartic functions | | |

In the ACARA syllabus, powers and polynomials are placed together. There seems no sense in this, however, since we have already dealt with $y = \frac{1}{x}$ and $y = \sqrt{x}$, and everything else to be covered are polynomials. Hence, we present here polynomials as a discrete unit.

Quartic functions are in the SACE syllabus, but not the ACARA syllabus. Thus, only South Australian students need study Section F.

It would be advantageous for students to have completed the 10A "Polynomials" chapter in the previous year.

## CHAPTER 5: THE UNIT CIRCLE AND RADIAN MEASURE

| | | SACE | ACARA |
|---|---|---|---|
| A | Radian measure | | |
| B | Arc length and sector area | | |
| C | The unit circle and the trigonometric ratios | Topic 3<br>Sub-topic 3.2 | Unit 1<br>Topic 2 |
| D | Applications of the unit circle | | |
| E | Multiples of $\frac{\pi}{6}$ or $\frac{\pi}{4}$ | | |
| F | The gradient of a straight line | | |

Having studied right angled triangle trigonometry since Year 9, we feel this material should be well known. A short review section will be placed online, but otherwise we want to get into more advanced trigonometry quickly.

It would be advantageous for students to have completed the 10A "Advanced trigonometry" chapter in the previous year.

Feedback over many years and from multiple curricula suggest that radian measure is something students find challenging. For me, this presents a case for introducing it earlier and applying it more often, rather than the other way around. We therefore present radians immediately, and continue working with both angle measures throughout.

By the time we consider the sine and cosine of multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$, we want students to have an ingrained radian mindset. Formulae are now stated in terms of radians only, and the equivalent ratios for degree measure are secondary.

In terms of the SACE curriculum, we present sub-topic 3.2 before sub-topic 3.1 so that students have calculated the cosine and sine of obtuse angles before meeting them in the context of the cosine and sine rules.

## CHAPTER 6: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

| | | SACE | ACARA |
|---|---|---|---|
| A | Areas of triangles | | |
| B | The cosine rule | Topic 3 | Unit 1 |
| C | The sine rule | Sub-topic 3.1 | Topic 2 |
| D | Problem solving | | |

This chapter is a fairly traditional presentation of these trigonometric applications. We include proofs for the rules where appropriate, and investigate the ambiguous case for the sine rule. The extensive problem solving section requires students to select the appropriate rule.

## CHAPTER 7: TRIGONOMETRIC FUNCTIONS

| | | SACE | ACARA |
|---|---|---|---|
| A | Periodic behaviour | | |
| B | The sine function | | |
| C | The cosine function | Topic 3 | Unit 1 |
| D | The tangent function | Sub-topic 3.3 | Topic 2 |
| E | Trigonometric equations | | |
| F | Using trigonometric functions | | |
| G | Angle sum and difference identities | | |

The general introduction to periodic functions provides a range of contextual examples.

We develop each component of the general sine function using the transformations we studied in Chapter 3. However, for the purpose of modelling, we think it is really important to stick the whole lot together. Otherwise you end up with a silly contextual model of a situation which is not real world at all. We have included questions involving more than one transformation, since this will prove useful for real world modelling. This is where the students learn their craft of identifying and utilising what parameters actually do in modelling. It is unclear whether the syllabus requires this, however.

We feel the SACE and ACARA syllabuses are a bit silly in using $\sin(x+c)$, $\cos(x+c)$, and $\tan(x+c)$ rather than (the equivalent) $\sin(x-c)$, $\cos(x-c)$, and $\tan(x-c)$, since this is directly counter to the work on transformations done previously. For this reason we stick to the traditional $\sin(x-c)$, etc. Once we have moved on to trigonometric functions, it is more important than ever that students are entrenched in the radian mindset. The whole notion of period can get very confusing if they are not thinking in radians.

For students who are also studying Specialist Mathematics, it is important that this chapter is completed in semester 1. Otherwise, they will not have the necessary background for the chapter in Specialist Mathematics which flows on from this one.

## CHAPTER 8: COUNTING AND THE BINOMIAL EXPANSION

| | | SACE | ACARA |
|---|---|---|---|
| A | The product and sum principles | | |
| B | Permutations | | |
| C | Factorial notation | Topic 4 | Unit 1 |
| D | Combinations | Sub-topic 4.1 | Topic 3 |
| E | Binomial expansions | | |
| F | The binomial theorem | | |

This chapter provides a general introduction to the principles of permutations and combinations, and explores the link between combinations and the binomial expansion.

Students following the ACARA syllabus, who are studying both Methods and Specialist Mathematics, may already have encountered permutations and

combinations in their Specialist course. These students may skip through Sections A to D, and focus their study on the binomial expansion in Sections E and F.

It is important to lay a solid foundation of understanding for the binomial expansion in Year 11, as this will hold students in good stead for their study of the binomial distribution in Year 12.

## CHAPTER 9: PROBABILITY

| | | SACE | ACARA |
|---|---|---|---|
| A | Experimental probability | | |
| B | Sample space | | |
| C | Theoretical probability | | |
| D | Compound events | | Unit 1 |
| E | Tree diagrams | | Topic 3 |
| F | Sets and Venn diagrams | | |
| G | The addition law of probability | | |
| H | Conditional probability | | |
| I | Independent events | | |

This material has been removed from the SACE syllabus. We find this somewhat surprising, seeing as students will encounter some fairly advanced probability concepts in Year 12, including probability distribution functions, expected values, binomial and Bernoulli distributions, and the normal distribution.

Students should be very familiar with sets from their work in the middle school years. As a result, sets are not studied in their own right in this book, but are considered in this chapter in the context of probability. The notions of Venn diagrams, intersection, and union are revised in the sets Section F. Students have been finding probabilities from Venn diagrams since Year 8, so they should not linger on this section. These concepts are then applied to the study of laws of probability, conditional probability, and independent events. Students who completed Year 10A in the previous year should have already encountered these ideas.

## CHAPTER 10: NUMBER SEQUENCES

| | | SACE | ACARA |
|---|---|---|---|
| A | Number sequences | | |
| B | Arithmetic sequences | Topic 7 | Unit 2 |
| C | Geometric sequences | Sub-topic 7.1, | Topic 2 |
| D | Series | 7.2 | |
| E | Arithmetic series | | |
| F | Geometric series | | |

This content is listed in the SACE syllabus as Topic 7, which falls under the banner of "preparation for Stage 2 Specialist Mathematics". However, we are placing it in the Methods textbook because it is listed in Mathematical Methods in the ACARA syllabus. This should not cause a problem for South Australian students, as students

preparing for Stage 2 Specialist Maths should have *both* the Year 11 Methods and the Year 11 Specialist textbook.

## CHAPTER 11: STATISTICS

| | | SACE | ACARA |
|---|---|---|---|
| A | Key statistical concepts | Topic 4 Sub-topic 4.2 | |
| B | Measuring the centre of data | Topic 4 Sub-topic 4.3 | |
| C | Measuring the spread of data | | |
| D | Variance and standard deviation | | |
| E | The normal distribution | Topic 4 Sub-topic 4.4 | |

Statistics is in the SACE syllabus at Year 11, but not the ACARA syllabus. Therefore, only South Australian students need to study this chapter.

Sections A and B are largely revision, looking at graphing discrete and continuous numerical data, and finding the centre of a data set. Students who are comfortable with these concepts should skip quickly through these sections.

Students who completed the 10A course should have briefly encountered standard deviation, however those who did not will be seeing it for the first time. Students are asked to calculate the sample standard deviation, first by hand, then using technology, as outlined in the syllabus. However, we also mention population standard deviation, as we feel it is important for students to be familiar with both concepts, and to understand the difference between them.

The chapter also contains an introduction to the normal distribution. It includes a discussion of how the normal distribution arises, and uses the '68 - 95 - 99.7%' rule to find the proportion of values within certain intervals. It provides a valuable grounding for the more advanced study of the normal distribution in Year 12.

## CHAPTER 12: SURDS, INDICES, AND EXPONENTIALS

| | | SACE | ACARA |
|---|---|---|---|
| A | Surds | | |
| B | Indices | | |
| C | Index laws | Topic 5 Sub-topic 5.1 | |
| D | Scientific notation | | |
| E | Rational indices | | Unit 2 Topic 1 |
| F | Algebraic expansion and factorisation | | |
| G | Exponential equations | | |
| H | Exponential functions | Topic 5 Sub-topic 5.2 | |
| I | Growth and decay | | |
| J | The natural exponential $e^x$ | | |

Sections A-D should be familiar to many of the students, and should be worked through quickly.

Rational indices may or may not be familiar to students, depending on whether they covered the 10A material in the previous year. The ability to write expressions involving surds in terms of rational indices is important in the lead-up to calculus, where students must differentiate functions like $f(x) = x\sqrt{x}$.

The natural exponential $e^x$ is not explicitly mentioned in the syllabus, but we have included it so that students meet both it and natural logarithms in Year 11. This is important because they need to be comfortable both graphing and performing differentiation with them in Year 12.

## CHAPTER 13: LOGARITHMS

| | | SACE | ACARA |
|---|---|---|---|
| A | Logarithms in base 10 | Topic 5<br>Sub-topic 5.3 | |
| B | Logarithms in base *a* | | |
| C | Laws of logarithms | | |
| D | Natural logarithms | | |
| E | Solving exponential equations using logarithms | | |
| F | Growth and decay | | |

As with statistics, logarithms are in the SACE syllabus at Year 11, but not the ACARA syllabus. Therefore, only South Australian students are required to study this chapter.

Students who completed 10A in the previous year would have encountered logarithms in base 10. However, students who did not take 10A will be seeing logarithms for the first time.

Natural logarithms are not in the syllabus, however we have included them because in Year 12, students will be dealing with natural logarithms more so than logarithms in other bases. We feel it is only sensible to introduce them in Year 11, so that students are familiar with them before needing to graph and differentiate with them in Year 12.

## CHAPTER 14: INTRODUCTION TO DIFFERENTIAL CALCULUS

| | | SACE | ACARA |
|---|---|---|---|
| A | Rates of change | Topic 6<br>Sub-topic 6.1, 6.2 | Unit 2<br>Topic 3 |
| B | Instantaneous rates of change | | |
| C | Finding the gradient of the tangent | | |
| D | The derivative function | | |

This chapter provides students with their first look at differential calculus.

We had originally planned to present limits first, so it did not interrupt the "flow" of the chapter as we moved from rates of change to the derivative function.

However, seeing as only differential calculus is studied in Year 11, only a very brief study of limits is required anyway. Therefore, we decided to lead with rates of

change, and introducing the $\delta y/\delta x$ notation. We use this to motivate a quick study of limits, where we only deal with limits where $x$ is finite.

Rates of change feature heavily in the syllabus, however we were keen to not get too laborious with them. Spending too much time on rates of change detracts from the power of calculus, where the whole point is that you can generate a function which allows us to quickly find the rate of change at any value of $x$.

We use Leibniz notation as in the syllabus as we introduce the derivative function, but are careful not to overdo it, thus avoiding unnecessary complexity.

## CHAPTER 15: PROPERTIES AND APPLICATIONS OF DERIVATIVES

| | | SACE | ACARA |
|---|---|---|---|
| A | Simple rules of differentiation | Topic 6 Sub-topic 6.3, 6.4 | |
| B | Tangents and normals | | Unit 2 |
| C | Increasing and decreasing functions | Topic 6 | Topic 3 |
| D | Stationary points | Sub-topic 6.5 | |
| E | Kinematics | | |
| F | Optimisation | | |

We have included the study of normal in with the study of tangents, as they link well with the students' previous work on perpendicular gradients.

The ACARA syllabus includes anti-derivatives in this topic, however we believe it is an inappropriate afterthought to stick anti-derivatives at the end of Year 11 without immediate extension to integral calculus. It is our view that spare time would be better spent reinforcing the differential calculus and its applications. The SACE syllabus has also excluded anti-derivatives for Year 11. We have therefore only discussed anti-derivatives in an Investigation at the end of the chapter