HAESE MATHEMATICS
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TEACHER NOTES FOR YEAR 12 MATHEMATICAL METHODS

CHAPTER 1: FUNCTIONS

		SACE	ACARA
A	Exponential functions	Topic 1 Sub-topic 1.3	Unit 3 Topic 1
B	Logarithms	Topic 4	Unit 4
C	Logarithmic functions	Sub-topics 4.1, 4.2	Topic 1
D	Trigonometric functions	Topic 1 Sub-topic 1.4	Unit 3 Topic 1

The main purpose of this chapter is for students to familiarise themselves with exponential, logarithmic, and trigonometric functions, before studying the calculus of these functions in later chapters.

Section B provides a brief recap of logarithms, on the basis that students have already encountered logarithms in Year 11. However, students following the ACARA syllabus may not have studied logarithms in Year 11. For this reason, an introduction to logarithms, based on the work in the Year 11 book, is provided as an online link. Students following the SACE syllabus, who completed the logarithms chapter in Year 11, should continue straight on to the logarithms section presented in the textbook.

Dealing with logarithms at this early stage allows us to include the derivatives and integrals of logarithmic functions in the calculus chapters, rather than having to address the calculus of logarithmic functions separately later.

CHAPTER 2: DIFFERENTIATION

		SACE	ACARA
A	First principles	Topic 1	
B	Simple rules of differentiation	Sub-topic 1.1	
C	The chain rule		Unit 3
D	The product rule	Sub-topic 1.2	Topic 1
E	The quotient rule		
F	Derivatives of exponential functions	Sub-topic 1.3	
G	Derivatives of logarithmic functions	Topic 4 Sub-topic 4.3	Unit 4 Topic 1
H	Derivatives of trigonometric functions	Topic 1 Sub-topic 1.4	Unit 3 Topic 1
I	Second derivatives	Sub-topic 1.5	

In this chapter we recap first principles and simple rules of differentiation from Year 11. Students are then introduced to the chain, product, and quotient rules, before differentiating the exponential, logarithmic, and trigonometric functions studied in Chapter 1.

The SACE syllabus appears to suggest performing applications (such as slopes of tangents, stationary points, and kinematics) for each function as the derivative is learnt. However, we believe it is more logical to present the applications later as a separate chapter, once all of the differentiation rules have been learnt. This way, each application can be taught just once, and then applied to all of the function types.

CHAPTER 3: APPLICATIONS OF DIFFERENTIAL CALCULUS

		SACE	ACARA
A	Equations of tangents	Topic 1	
B	Increasing and decreasing functions	Sub-topics	
C	Stationary points	1.1, 1.3, 1.4	Unit 3
D	Inflections and shape	Sub-topic 1.5	Topic 1
E	Kinematics	Sub-topics	
F	Rates of change	1.1, 1.3, 1.4, 1.5	
G	Optimisation		

In this chapter we look at the applications of differential calculus. The chapter gives students the opportunity to practise differentiating a wide variety of functions.

Many of the concepts explained in this chapter will be familiar to students, as they were studied in Year 11. The focus this year is in applying the concepts to more complicated functions.

Students also have the chance to explore the surge function and the logistic function in real-world contexts.

CHAPTER 4: INTEGRATION

		SACE	ACARA
A	The area under a curve	Topic 3 Sub-topic 3.2	
B	Antidifferentiation	Sub-topic 3.1	
C	The Fundamental Theorem of Calculus	Sub-topic 3.3	Unit 3
D	Integration		Topic 2
E	Rules for integration	Sub-topic 3.1	
F	Integrating $f(ax+b)$		
G	Definite integrals	Subtopics 3.2, 3.3	

We now move to SACE Topic 3: Integral Calculus. The reason for this is that students studying both Methods and Specialist Mathematics must complete all of the calculus in Methods *before* starting the calculus in Specialist Mathematics. To help achieve this, all of the calculus in the Methods textbook has been presented as early as possible.

CHAPTER 5: APPLICATIONS OF INTEGRATION

		SACE	ACARA
A	The area under a curve	Topic 3	
B	The area between two functions	Sub-topics 3.2, 3.4	Unit 3 Topic 2
C	Kinematics	Sub-topic 3.4	
D	Problem solving by integration		

We now explore some applications of integration, including the area under and between curves, kinematics, and problem solving. We now formally establish that for functions $f(x) \leq 0$, we must negate the integral to find the area between $f(x)$ and the x-axis.

It is at this point that the Methods and Specialist classes will need to be well coordinated. Chapter 7 of the Specialist textbook follows directly on from this chapter, so this chapter must be completed before the Specialist classes reach Chapter 7.

CHAPTER 6: STATISTICS

A Key statistical concepts
B Measuring the centre of data
C Variance and standard deviation

This chapter gives students the opportunity to revise some important statistical concepts.

Students following the ACARA syllabus did not do any statistics in Year 11, and it is possible that they have never encountered variance and standard deviation. It is important that these students understand the basic ideas behind variance and standard deviation, before they encounter more advanced treatments of these measures of spread in later chapters.

South Australian students can skip Sections A and B, as these were completed in Year 11. Section C is different from what was presented in Year 11, and it gives students another chance to familiarise themselves with variance and standard deviation. The material in this Section prepares students for how variance and standard deviation is approached in the coming chapters, with a greater emphasis on population standard deviation and variance, including from a frequency table.

CHAPTER 7: DISCRETE RANDOM VARIABLES

		SACE	ACARA
A	Random variables		
B	Discrete probability distributions	Topic 2	
C	Expected value	Sub-topic 2.1	Unit 3
D	Variance and standard deviation		Topic 3
E	Properties of $aX+b$		
F	The Bernoulli distribution	Sub-topic 2.2	
G	The binomial distribution	Sub-topic 2.3	

This SACE Topic 2 chapter has been pushed back, so that the calculus chapters can be completed as early as possible.

Students should recognise that the Bernoulli random variable is a special case of the binomial random variable, where the trial is performed only once. The relationship between these variables will be further developed in Chapter 9.

CHAPTER 8: CONTINUOUS RANDOM VARIABLES

		SACE	ACARA
A	Continuous random variables	Topic 5	
B	Probability density functions	Sub-topic 5.1	
C	The normal distribution		Unit 4
D	Probabilities using a calculator	Sub-topic 5.2	Topic 2
E	The standard normal distribution (Z-distribution)		
F	Quantiles		

When defining the mean and standard deviation of a continuous random variable, the syllabus describes an integral over the infinite domain from $-\infty$ to ∞. However, in the overwhelming majority of cases the students will encounter, continuous random variables are defined over a finite domain $[a, b]$, and we have defined the mean and standard deviation accordingly. It seems inappropriate to provide these definitions in a form that the students do not have the tools to evaluate. The right hand column of the SACE syllabus even states that integrals of this form should be found using technology, with sufficiently large values to approximate the end points. We have included some questions with infinite domains which require students to do this.

CHAPTER 9: SAMPLING AND CONFIDENCE INTERVALS

		SACE	ACARA
A	Sampling distributions	Topic 5	
B	Distribution of sample means	Sub-topic 5.3	
C	The Central Limit Theorem		
D	Confidence intervals for means	Topic 6 Sub-topic 6.1	
E	Sample proportions	Sub-topic 6.2	Unit 4
F	Confidence intervals for proportions	Sub-topic 6.3	Topic 3

We have placed SACE Sub-topic 5.3 (Sampling) in this chapter, as it appears to fit more sensibly with the material in Topic 6 than with the rest of Topic 5.

When presenting the Central Limit Theorem, we state that $n = 30$ is used as a "rule of thumb" to decide whether a sample size is large enough for the distribution of sample means to be approximately normal. However, students should understand that this is a guide only, and there is no "magical threshold" at which distributions change from being non-normal to normal; it is a gradual process through which the approximation to normality improves as the sample size increases.

For students following the ACARA syllabus, the work on statistical inference for sample means is actually part of the Specialist Mathematics course. This should not be a problem, as students studying Specialist Mathematics will also be studying Mathematical Methods, and so they should have both textbooks. If possible, it would be best if these students complete the material on sample means and sample proportions together rather than separately, as the ideas involved are very similar, and the results from both components each stem from the Central Limit Theorem in Section C.

Students following the ACARA syllabus who are studying Mathematical Methods only, need only complete Sections E and F. That being said, their understanding of the work will be enhanced by studying some relevant aspects of the earlier sections, especially the Central Limit Theorem in Section C.