Mathematics for Australia Year 6 2nd edition

Chapter summaries

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CHAPTER 1: WHOLE NUMBERS

- **A** Place value
- **B** Number lines
- **C** Big numbers
- **D** Rounding numbers

Keywords:

- billion
- counting number
- digit
- expanded form
- greater than
- Hindu-Arabic number system
- less than
- million
- natural number
- number line
- number system
- numeral
- numeral form
- place value
- place value chart
- round
- round down
- round up
- trillion
- whole number
- zero

In this second edition, the opening chapter focuses on the structure of whole numbers, rather than performing operations with them. The operations work has been moved to Chapter 2, where it is combined with the words associated with operations that was previously covered in Chapter 3 (Number properties).

We describe the expanded form of the number 6794 as simply $6000+700+90+4$ rather than $6\times1000+7\times100+9\times10+4$. This is in part because the students have not done order of operations yet, so the second expression could be confusing. There is a greater emphasis on students converting numbers between numeral form and word form.

In Section B (Number lines), the emphasis is on using the number line to order numbers, rather than to perform operations with the numbers. We take the opportunity to introduce students to the $<$ and $>$ signs.

In Section C (Big numbers), students are introduced to millions, billions, and trillions at the same time. We feel that they are not conceptually different, and that it is easier for the student to see them together.

In Section D (Rounding numbers), the term *multiple* is used, even though multiples are not covered until Chapter 4. Here, we only consider multiples of 10, 100, and 1000, and the terms are explained as they are needed.

CHAPTER 2: OPERATIONS

- **A** Addition
- **B** Subtraction
- **C** Multiplication
- **D** Column multiplication
- **E** Division
- **F** Problems with multiple operations
- **G** Index notation
- **H** Order of operations

Keywords:

- add
- addition
- base
- BEDMAS
- difference
- divide
- dividend
- division
- divisor
- exponent
- index
- index notation
- multiplication
- multiply
- operations
- power
- product
- quotient
- remainder
- subtract
- subtraction
- sum

This is a new chapter for this second edition. It combines the material on operations with whole numbers which previously existed in Chapter 1 (Whole numbers), with the words associated with operations, index notation, and order of operations which previously existed in Chapter 3 (Number properties). We believe it is more sensible to have all of this material together in the one chapter.

Rather than simply presenting strategies for performing operations, we have presented investigations aimed at getting students to discover the strategies for themselves. We think this will help them appreciate and understand why the strategies are helpful. We have also presented the strategies *before* giving the standard techniques using columns, as students should first try to use the strategies to perform operations quickly. If no such strategy is appropriate, only then should they use a column method.

The multiplication and division by powers of 10 have been separated into the multiplication and division sections, since they are important multiplication and division strategies in their own right. In particular, the ease of multiplying by a power of 10 forms the basis for other multiplication strategies.

CHAPTER 3: LINES AND ANGLES

- **A** Lines
- **B** Angles
- **C** Measuring angles
- **D** Calculating angles
- **E** Vertically opposite angles

Keywords:

- acute angle
- angle
- arms
- bracket notation
- degrees
- intersecting lines
- line
- line segment
- obtuse angle
- parallel lines
- point
- point of intersection
- protractor
- ray
- reflex angle
- revolution
- right angle
- straight angle
- three point notation
- vertex
- vertically opposite angles

We have organised the sections of this chapter into a more logical structure. The opening section is primarily about lines rather than points, so we have moved the material about points before Section A, and Section A is now named "Lines" rather than "Points and lines".

We feel it is better that students are familiar with how to describe and classify an angle before they measure angles, so Section B now only asks students to name and classify angles. Measuring by protractor is then covered in Section C.

Section D (Calculating angles) incorporates what was previously in Section 2C (Angles at a point or on a line), as well as the questions from Section 2B (Angles) that dealt with finding unknown angles by addition or subtraction of known angles. We feel these questions fit well together under the title of "Calculating angles", as the strategies employed are very similar. Notice that some questions require students to find x, but we are not expecting students to formally solve equations at this stage. The students should find x intuitively, using the information given about right angles, angles on a line, or angles at a point.

CHAPTER 4: NUMBER PROPERTIES

- **A** Zero and one
- **B** Square numbers
- **C** Cubic numbers
- **D** Triangular numbers
- **E** Divisibility
- **F** Divisibility tests
- **G** Factors
- **H** Prime and composite numbers
- **I** Highest common factor
- **J** Multiples

Keywords:

- composite number
- cubic number
- divisible
- even
- factor
- factor pair
- highest common factor
- identity
- multiple
- odd
- one
- prime number
- square number
- triangular number
- undefined
- zero

All of the operations work that previously existed in this chapter has been moved to Chapter 2 (Operations).

In the Discussion in Section A, students should find that $0 + 12 = 12$, $0 \times 12 = 0$, and $1 \times 12 = 12$. Subtraction and division do not have this property, since for example $5 - 3$ does not equal $3 - 5$, and $5 \div 3$ does not equal $3 \div 5$. The special name for this property is commutativity.

In this edition, we have mentioned cubic numbers as well as square and triangular numbers, as it is useful to introduce cubic numbers in the same way as square and triangular numbers are introduced.

We have introduced highest common factor, even though it is not mentioned in the syllabus at Year 6, so we can use it for describing what happens when reducing fractions to lowest terms. We talk about multiples, but we will not introduce lowest common multiple yet, since addition and subtraction of fractions is only done where one denominator is a multiple of the other. Lowest common multiples will be covered in Year 7.

CHAPTER 5: GEOMETRIC SHAPES

- **A** Polygons
- **B** Triangles
- **C** Quadrilaterals
- **D** Circles
- **E** Solids
- **F** Drawing solids
- **G** Nets of solids

Keywords:

- apex
- circle
- closed figure
- compass
- cone
- cross-section
- cube
- cylinder
- edge
- equilateral triangle
- face
- irregular polygon
- isosceles triangle
- kite
- net
- parallelogram
- polygon
- prism
- pyramid
- quadrilateral
- radius
- rectangle
- regular polygon
- rhombus

- scalene triangle
- solid
- solid of uniform cross-section
- sphere
- square
- surface
- three-dimensional
- trapezium
- triangle
- two-dimensional
- vertex

Renaming the chapter to "Geometric shapes", as opposed to "Polygons and solids" in the previous edition, allows us to include a section about circles. This is useful for describing the cross-section of a cylinder and the base of a cone later in the chapter.

Section F (Drawing solids) has been moved here from Year 7, as we feel it is a fundamental aspect of solids. The views of solids, as well as the oblique and isometric projections, remain in Year 7. We have taken the opportunity here to introduce the word *ellipse*, to describe the shape of a circle viewed from an angle. This shape is used when drawing cones, spheres, and cylinders.

CHAPTER 6: FRACTIONS

- **A** Fractions
- **B** Fractions as division
- **C** Proper and improper fractions
- **D** Fractions on a number line
- **E** Equal fractions
- **F** Lowest terms
- **G** Comparing fractions
- **H** Adding and subtracting fractions
- **I** Multiplying a fraction by a whole number
- **J** A fraction of a quantity

Keywords:

- bar
- denominator
- equal fractions
- fraction
- improper fraction
- lowest terms
- mixed number
- numerator
- proper fraction
- simplest form

In Section C, we have simplified the process of converting between improper fractions and mixed numbers.

Having introduced highest common factor in Chapter 4, we can now use this term when describing how to reduce fractions to lowest terms.

We have tried to avoid the term "simplest form" as much as possible in this new series, as it can be quite ambiguous. The simplest form to work with quite often depends on the context, and what you are trying to do. So, we have predominantly used "lowest terms" to describe fractions. However, we have kept the simplest form usage occasionally, as we understand this is in common use.

In the Discussion of an alternative method of adding and subtracting fractions in Section H, students should find that this approach is most useful when proper fraction parts sum to less than one. They should also notice that care must be taken when using this method to subtract fractions, since the fraction part of the subtracted fraction must also be subtracted.

We have added Section I (Multiplying a fraction by a whole number), so that students can find fractions of quantities more sensibly using fraction multiplication, rather than division. All that is required is for students to recognise that "of" means that we multiply.

CHAPTER 7: DECIMALS

- **A** Decimal numbers
- **B** Decimal numbers on a number line
- **C** Ordering decimal numbers
- **D** Rounding decimal numbers
- **E** Converting decimals to fractions

- **F** Converting fractions to decimals
- **G** Adding and subtracting decimal numbers
- **H** Multiplying by powers of 10
- **I** Dividing by powers of 10
- **J** Multiplying decimals by a whole number
- **K** Dividing decimals by a whole number

Keywords:

- decimal number
- decimal point
- hundredth
- place value
- tenth
- thousandth

In the Discussion in Section A, students should recognise that including a "0" before the decimal point in 0.37 makes the decimal point stand out more. Students should see that 1.5 and 1.50 have the same value, and there are some situations, such as dealing with currency, where we would write 1.50 rather than 1.5 . It may be worth returning to this Discussion once students have completed the section on rounding decimal numbers, and have students think about the difference between rounding 1.5032 to 1.5 (one decimal place) or 1.50 (2 decimal places). Is the second approximation more accurate, even though both approximations have the same value?

In Section C, we extend the work in Section B and use a number line to order decimal numbers, in line with what has been done with whole numbers and fractions. We then look at a more formal method for ordering decimals.

In the Discussion in Section D, students should indeed find that the same rule for rounding whole numbers can also be used to round decimal numbers.

When multiplying and dividing decimal numbers by powers of 10, students should understand that each digit moves a particular number of places in the place value chart. This has the *effect* of moving the decimal point.

We have updated the method used to multiply a decimal number by a whole number. Students should be encouraged to think about the place values involved, as indicated by the worked example. This will help students understand where the decimal point should be placed.

CHAPTER 8: MEASUREMENT: INTRODUCTION

- **A** Units
- **B** Reading scales
- **C** Mass

Keywords:

- gram
- kilogram
- mass
- milligram
- scale
- tonne
- units

We wanted to give students a more general introduction to measurement, so we have added this chapter, which talks students through the different units used to measure things, and how to read scales. This chapter contains plenty of Activities for students to explore different measuring devices, and provides students the opportunity to think about when we measure things in our daily lives, and why different units of measurement are used.

The Discussion in Section A is a prelude to the distinction between discrete and continuous variables in later years. Students should find that is does not make sense to use fractions or decimals when counting items (you cannot have $\frac{1}{2}$ a car or 4.5 people), but it does make sense when measuring quantities (you can have $\frac{1}{2}$ a metre or 4.5 kg).

CHAPTER 9: MEASUREMENT: LENGTH

- **A** Units of length
- **B** Operations with lengths
- **C** Perimeter
- **D** Scale diagrams

Keywords:

- centimetre
- kilometre
- length
- metre
- millimetre
- perimeter
- scale
- scale diagram
- scale factor

We have separated the work which was previously a single Section "Length" into two sections. In Section A students solely deal with converting between units of length, and in Section B students perform operations with lengths. This separation gives us the chance to highlight the idea that two lengths must be written in the same units before we can perform operations with them.

In the Discussion at the start of Section A, students should conclude that using parts of the body to measure length has problems because these body parts will differ in length from person to person.

In the Discussion at the start of Section B, it should be emphasised to students that we need to add units of the same type in order to obtain a meaningful answer. A good way to illustrate this would be to consider adding 2 metres and 10 centimetres. If we simply add the 2 and the 10, in what units should the answer be written?

We have now included Scale diagrams in this chapter. In the previous edition, the idea of scale was not introduced until the Location chapter. Including it here allows students to practise length conversions with it, and provides many more contexts to present scale diagrams, beyond maps on a coordinate grid.

CHAPTER 10: MEASUREMENT: AREA, VOLUME, AND CAPACITY

- **A** Area
- **B** The area of a rectangle
- **C** The area of a triangle
- **D** Volume
- **E** The volume of a rectangular prism
- **F** Capacity
- **G** Connecting volume and capacity

Keywords:

- area
- capacity
- cubic centimetre
- cubic metre
- cubic millimetre
- kilolitre
- litre
- megalitre
- millilitre
- square centimetre
- square kilometre
- square metre
- square millimetre
- volume

In this edition, we have minimised the amount of work done on "square units", in favour of getting students more quickly into using the metric units they would see around them such as square centimetres and square metres.

In the Discussion at the end of Section A, students should find that they can more quickly find the total area by multiplying the number of rows by the number of columns, rather than counting individual squares. This leads into the formula for finding the area of a rectangle in the next section.

We have included the area of a triangle in this edition, although it is not mentioned in the syllabus, as students will have already covered the area of a rectangle in Year 5, and we feel it is important that they have something new to consider in Year 6.

This may be the first time students have encountered capacity, so we have split the capacity material into two sections. Section F deals with the units of capacity and conversions between them, and Section G explores the connection between the units of capacity and volume.

CHAPTER 11: TIME

- **A** Time lines
- **B** Units of time
- **C** The calendar year
- **D** Time calculations

E 24-hour time
F Timetables

Keywords:

- 12-hour time
- 24-hour time
- calendar year
- day
- hour
- leap year
- minute
- month
- second
- time line
- timetable
- week
- year

In this edition, we have introduced a new Section C (The calendar year), which covers the number of days in each month, and explains the conditions for a leap year. This allows us to separate the time calculations involving dates, such as "How many days is it from April 24th to July 17th?", from the calculations involving times of the day such as "Find the time which is 3 hours 40 minutes after 10:45 am". We feel this is appropriate, as they are conceptually quite different.

In the Discussion at the start of Section B, students should discuss how people talk about time differently for longer periods such as days, weeks, or years. When we talk about the duration of events using these units, we usually do not mean that the task will be performed during this *entire* time period, rather that the time actually describes how long it will be between the task starting and finishing.

In Section E, students should be reminded to take care with times between 12 pm and 1 pm. These are pm times, but do not "change" when converted to 24-hour time.

CHAPTER 12: PERCENTAGE

A Percentage
B Converting percentages into fractions
C Converting fractions into percentages
D Converting percentages into decimals
E Converting decimals into percentages
F Number lines
G Expressing one quantity as a percentage of another
H Finding a percentage of a quantity
I Discount

Keywords:

- discount
- marked price
- percent
- percentage
- selling price

We have introduced Section F, which asks students to place percentages on a number line, in line with what was done in the Fractions and Decimals chapters. The focus here is on placing a mixture of fractions, decimals, and percentages on a number line by converting them all to decimals.

In Section H, students should be reminded of the method used to find fractions of quantities in Chapter 6. The updated method in Chapter 6 now ties in well with the method used here. Students should remember that "of" means that we multiply, and the only difference here is that we are working with a percentage, which we first convert to a decimal.

The material on Discount has now been made into a section of its own (Section I), rather than being part of Section H.

CHAPTER 13: POSITIVE AND NEGATIVE NUMBERS

A The number line
B Ordering numbers
C Words indicating positive and negative
D Addition and subtraction with negative numbers
E Adding and subtracting negative numbers
F Multiplying negative numbers
G Dividing negative numbers

Keywords:

- negative number
- negative sign
- opposite
- positive number
- positive sign

This chapter has been significantly restructured from the previous edition.

We have introduced the number line from the start of the chapter, as we feel that this will help students understand the relationship between positive and negative numbers.

We have also made clearer the distinction between talking about *position* (for example, representing 3 levels below ground level as -3), and *movements* (for example, representing going down 3 levels as "subtract 3"). In the contextual problems, students should understand that we start from a position on a number line, described by a positive or negative number, interpreted by the context. Then, there is a series of movements, corresponding to operations (addition or subtraction) involving positive quantities. We then end up at our final position, which may be positive or negative, to be interpreted in context. Understanding of all of this is set up by the Activity which distinguishes between positions and directions.

In the Discussion in Section A, students should find that the opposite of zero is zero.

In the Discussion in Section B, students should see that terms such as *larger* and *smaller* are potentially confusing when discussing positive and negative numbers, because the terms have an association with *size*. For example, -5 is less than -3, however some may argue that -5 is a *larger* number than -3, because it is further from 0.

We introduce multiplication and division by negatives here, even though it is not in the syllabus, as it is covered only very briefly in Year 8. As such, we felt that students should have a couple of years of studying it properly.

Multiplication by negative numbers can be conceptually difficult for many students, especially the idea of multiplying two negative numbers together. The Discussion at the start of Section F should be used to lead students in the right direction. By considering 3×-5 as "3 lots of -5", students should be able to see that $3 \times -5 = -15$. From there students could be asked to consider -3×-5 as "the opposite of 3×-5", which is the opposite of -15, which is 15. This should give students a justification that the product of two negative numbers is positive.

CHAPTER 14: SEQUENCES

- **A** Generating a sequence
- **B** Finding a rule for a sequence
- **C** Patterns

Keywords:

- pattern
- rule
- sequence

Rather than separating the content of this chapter into sequences involving whole numbers, then fractions, then decimals, in this edition we have first dealt with generating sequences based on a rule, and then considered finding the rule given members of the sequence. This allows us to give general instruction about each of these skills, which can be applied regardless of whether the sequence involves whole numbers, fractions, or decimals.

When generating sequences involving fractions (such as "Start at 2, and add $\frac{3}{8}$ each time"), students should be reminded that it is not always sensible to write the fractions in the sequence in lowest terms immediately, as it will be easier to generate the terms if the fraction is left with the same denominator as the fraction specified in the rule. The terms of the sequence may be written in lowest terms as the final answer.

When trying to find a rule for a given sequence, students should make sure their rule holds for *all* terms of the sequence, not just the first two terms. For example, for a sequence starting $2, 6, ...$, the rule may involve adding 4 each time, or it may involve multiplying by 3 each time. Subsequent terms of the sequence will need to be considered to determine the correct rule.

We conclude by asking students to describe rules for sequences generated by geometric patterns. Formulae is not mentioned in the new syllabus, so this work has been removed from this edition.

CHAPTER 15: LOCATION

- **A** Grid references
- **B** Finding points
- **C** Coordinates

D Positive and negative coordinates
E Compass points

Keywords:

- axes
- coordinates
- east
- grid reference
- map
- north
- northeast
- northwest
- number plane
- ordered pair
- origin
- south
- southeast
- southwest
- west
- x-axis
- x-coordinate
- y-axis
- y-coordinate

When dealing with coordinates, we have removed the idea of "movement from the origin" in order to plot points. Students should instead be encouraged to think of the axes as number lines, and to plot points on the number plane in the same way they would plot numbers on a number line. This is especially helpful when dealing with negative coordinates in Section D, as it removes the need to think about an x-coordinate of -2 as "moving 2 to the left".

In the Discussion in Section C, students could consider that using numbers makes it easier to compare the position of points relative to each other than with letters. A possible disadvantage of using numbers in both axes is that there is the potential to swap the coordinates, so the position $(3, 5)$ could be mistaken for $(5, 3)$.

Having introduced scales in Chapter 9, there is no need for a separate section about it here, and questions involving scale are included in Sections D and E.

The Discussion at the end of the chapter is intended to highlight the distinction between how directions have been used in this chapter, and how they are used in everyday language. The directions in this chapter have been used very precisely, so as to allow students to describe the exact coordinates of locations. However, in everyday language we might say that B is "to the south of A", even if it is not *due* south of A.

CHAPTER 16: PROBABILITY

A Describing probability
B Using numbers to describe probabilities
C Outcomes
D Calculating probabilities
E Complementary events

Keywords:

- 50-50 chance
- certain
- complementary events
- equally likely
- event
- impossible
- likely
- outcome
- probability
- unlikely

In this edition we have improved the number line, to better show the link between words associated with probabilities, and their corresponding probabilities as a number from 0 to 1.

In the Discussion at the end of Section A, students should be encouraged to think about what it means for an event to be *impossible* to occur, as opposed to "highly unlikely". Many events are extremely unlikely to occur, but unless we can definitively rule out the possibility of them occurring, we cannot classify them as impossible.

Section D ends with an Activity about experimental probability. This experiment has been deliberately chosen to be one for which theoretical probabilities can be calculated. Students should see that, as the number of trials of the experiment increases, the experimental probabilities get closer to the theoretical probabilities. This concept will be explored further in subsequent years.

Complementary events has been included in this edition, even though it is not explicitly mentioned in the syllabus. This is particularly included for students who used our Year 5 book in the previous year, as we wanted to extend the material here beyond what was done in Year 5.

CHAPTER 17: STATISTICS

- **A** Categorical data
- **B** Dot plots
- **C** Pictographs
- **D** Column graphs
- **E** Pie charts
- **F** Comparing categorical data
- **G** Numerical data
- **H** Measuring the centre of a data set

Keywords:

- average
- categorical data
- column graph
- data
- dot plot
- frequency
- mean
- mode
- numerical data
- pictograph
- pie chart
- side-by-side column graph
- statistics
- tally
- tally and frequency table

In the second edition, each type of graph for categorical data gets its own section. Pictographs has been moved earlier to just after dot plots. We did this because:

- We feel that pictographs follow on quite naturally from dot plots.
- This will be the last year they will do pictographs. So we do not want to linger on them any longer than necessary.

The Activity at the end of Section C allows students to practise recording their own data, organising it, and then drawing graphs to display their data. Please note that the bird watching simulation shows the birds in a random order, so each student will end up with a different data set. As an extension, you can ask the students to draw different types of graphs to display their data and discuss the effectiveness of each graph for conveying information.

In Example 6, we made sure that each bouquet has the same number of flowers. However, students should be aware that if the data sets are *different* sizes, then percentages should be used to compare categories, instead of frequencies. This is highlighted in the Discussion questions and in question **2** of Exercise 17F.

We have renamed the last section from "Mean or average" to "Measuring the centre of a data set" to better emphasise the *purpose* of the mean, and to be consistent with corresponding sections in later year levels. The content of this section however, has not changed very much from the first edition.

CHAPTER 18: TRANSFORMATIONS

- **A** Translations
- **B** Reflections
- **C** Rotations
- **D** Combinations of transformations

Keywords:

- anticlockwise
- centre of rotation
- clockwise
- image
- mirror line
- object
- reflection
- rotation
- transformation
- translation

In this edition, we have made the shadings of the object and the image the same, rather than having the object unshaded and the image shaded. We have done this to make sure that the only difference between the object and image is the transformation it has undergone.

In the Discussion at the start of Section C, students should find that, when a point is rotated about O, its distance from O does not change. This should help students when performing rotations of their own.

In the Discussion at the end of Section C, students should find that a translation only changes the position of an object, while a reflection and rotation also change the orientation of the object. Most importantly, none of these transformations change the size or shape of the object. This will be emphasised further in coming years as we start to talk about congruent figures.