MATHEMATICS 9 MYP 4 third edition

Chapter summaries

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ASSUMED KNOWLEDGE 1: NUMBER

A Prime and composite numbers
B Highest common factor
C Multiples
D Operations with fractions
E Rounding numbers
F Roots
G Rational numbers

Keywords:

- common fraction
- composite number
- cube root
- decimal places
- denominator
- equal fractions
- factor
- fraction
- highest common factor
- improper fraction
- lowest common denominator
- lowest common multiple
- mixed number
- multiple
- numerator
- place value
- prime number
- proper fraction
- radical
- rational number
- reciprocal
- recurring decimal
- round
- significant figures
- square root
- terminating decimal

This online chapter has been included for students who feel they need extra help with the basic properties of numbers. Students who need this help should be encouraged to study the chapter out of classroom time, since most students will not need this chapter, and classroom time spent on this chapter is likely to come at the expense of time spent on the more advanced chapters at the end of the year.

ASSUMED KNOWLEDGE 2: PERCENTAGE

A Converting percentages into decimals and fractions
B Converting decimals and fractions into percentages
C Expressing one quantity as a percentage of another
D Finding a percentage of a quantity

Keywords:

- percentage

This online chapter is intended for students who need help with the basics of percentages. This chapter essentially revises the work from Sections A to D of the Percentage chapter (Chapter 5) in MYP 3. Percentages will be used throughout this book, especially in Chapter 11 (Financial mathematics), so any student who feels that they do not have a good grasp of percentages should be encouraged to complete this chapter. As with Assumed Knowledge 1, classroom time should not be spent on this chapter.

CHAPTER 1: NUMBER

A Exponent notation
B The Fundamental Theorem of Arithmetic
C Order of operations
D Absolute value

Keywords:

- absolute value
- base
- BEDMAS
- counting number
- exponent
- exponent notation
- factor tree
- index
- integer
- natural number
- power
- prime factored form
- prime factorisation
- prime number
- real number
- repeated division
- whole number

This chapter has been added to the book for this new edition. Section A largely covers what was in the start of Chapter 2 (Indices) in the previous edition. We now prefer to use the term "exponent" rather than "index", as this is the terminology used in the MYP Framework.

Given that there is no dedicated fractions chapter at MYP 4, we also have the opportunity to revise the order of operations here, without the need to leave the fractions material until a later chapter. This material should be familiar to most students, and may be skipped if this is the case.

The inclusion of this Number chapter allows us to study absolute value, which will be used in later years.

CHAPTER 2: ALGEBRA: EXPRESSIONS

A Algebraic notation
B Writing expressions
C Algebraic substitution
D The language of algebra
E Collecting like terms
F Algebraic products
G Algebraic quotients
H Algebraic common factors

Keywords:

- algebra
- algebraic fraction
- algebraic product
- algebraic quotient
- coefficient
- constant
- equation
- evaluate
- exponent notation
- expression
- highest common factor
- like terms
- product notation
- substitute
- term
- variable

This chapter has been adapted from what was Chapter 1 (Algebra) in the previous edition. In this edition, Section D (The language of algebra) has been included to reinforce the terminology introduced in MYP 3.

In the Discussion at the end of Section B, students should find that the difficulty in talking about the difference between 5 and x is that, without knowing the value of x, we do not know which number is greater. So, the difference may be $x - 5$, or $5 - x$. By writing the difference between 5 and x as $|5 - x|$, we can avoid this difficulty and guarantee that our difference is positive.

In this edition, we have added a section on algebraic quotients (Section G). In this section, we simplify algebraic quotients by cancelling common factors, without the use of the exponent laws. This provides a lead-in to the more involved simplifications and operations done in Chapter 15 (Algebraic fractions).

Algebraic common factors have been added to the end of this chapter to mirror what was done in MYP 3. This work will be useful when factorising algebraic expressions.

CHAPTER 3: EXPONENTS

- **A** Exponent laws
- **B** Zero and negative exponents
- **C** Standard form (scientific notation)
- **D** International system (SI) units

Keywords:

- derived unit
- exponent laws
- International System of Units
- negative exponent law
- scientific notation
- standard form
- zero exponent law

In this edition, we have changed the name of this chapter from "Indices" to "Exponents" to reflect the terminology in the MYP Framework.

"Evaluating indices" has been moved to Chapter 1 (Number), and is now called "Exponent notation". This appears to be a more sensible place to introduce exponent notation, given that expressions involving exponents are presented in both Chapters 1 and 2.

In the Discussion at the end of Section B, students should realise that cancelling the common factor of x is problematic if $x = 0$, since we cannot divide by 0. Students should be able to use the exponent laws to show that $\frac{x}{x} = x^0$. Given that any non-zero number divided by itself is 1, this is reasonable justification for concluding that $a^0 = 1$ for $a \neq 0$. However, when $x = 0$, we have $\frac{0}{0} = 0^0$. Since $\frac{0}{0}$ is undefined, this leads us to the conclusion that 0^0 is undefined.

We have adjusted our approach to explaining standard form, to give students more guidance in how to choose their values of a and k in $a \times 10^k$.

Section 2D (Rational indices) has been removed in this edition, since it is only part of the Extended content in the Framework. We have instead added a new section "International System (SI) units", in which we use prefixes such as "micro", "nano", "mega", and "giga", in conjunction with standard form, to write very large and very small quantities.

CHAPTER 4: ALGEBRA: EXPANSION

- **A** The distributive law
- **B** The product $(a + b)(c + d)$
- **C** The difference between two squares
- **D** The perfect squares expansion
- **E** Further expansion

Keywords:

- difference between two squares
- distributive law
- expansion
- FOIL rule
- perfect squares

In this edition, much more of this content will be explored for the first time. We have removed quadratic algebraic factorisation from MYP 3 in this edition, since the most recent Framework specifies that this should not be introduced until MYP 4. It therefore seemed sensible to remove the corresponding work on algebraic expansion at MYP 3. As a result, students will have only seen the distributive law at MYP 3, and the rest of the expansion work will be new at MYP 4.

In response to this, we have increased the number of questions in this chapter, in order to give students more practice.

Students should be encouraged to see the difference between two squares expansion, and the perfect squares expansion, as special cases of the $(a + b)(c + d)$ expansion. Questions **3** and **4** of Exercise 4B should help with this.

In Activity 2 at the end of Section D, students should find that:

- The sum of the squares of any two positive odd integers will be even, but not a multiple of 4.
- If the sum of the squares of two integers is odd, then one of the integers is odd, and the other is even.

In Section E (Further expansion), it may help students to recognise that they have already encountered the idea of "multiplying each term in the first bracket by each term in the second bracket", since that is what happens when applying the FOIL rule to $(a+b)(c+d)$ in Section B.

CHAPTER 5: SETS

- **A** Sets
- **B** Complement of a set
- **C** Intersection and union
- **D** Special number sets
- **E** Interval notation

Keywords:

- complement
- complementary set
- disjoint
- element
- empty set
- equal sets
- infinite set
- integer
- intersection
- interval notation
- irrational number
- member
- natural number
- negative integer
- positive integer
- rational number
- real number
- set
- set notation
- subset
- union
- universal set

In this edition, we have split the Sets and Venn diagrams chapter into two separate chapters. In the previous edition, students were asked to solve linear equations when finding unknown numbers of elements in Venn diagrams, before they reached the linear equations chapter.

We felt the best approach was to split Sets and Venn diagrams into two chapters, with "Linear equations and inequalities" in between them. This way, interval notation can be introduced in Sets before it is used in linear inequalities, and linear equations can be used to solve problems involving Venn diagrams.

We have introduced the concept of sets more slowly in this edition. For example, rather than including intersection and union in Section A, intersection and union are presented in their own section in Section C. This allows students to become comfortable with the concepts and terminology of sets in Section A, before we perform operations with them.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

In the Discussion at the end of Section C, it would be helpful for students to experiment with cases where one of the unknown numbers is a multiple of the other, and cases where neither number is a multiple of the other. Students should determine that there will always be sufficient information to determine the values of the two numbers. They should find that the largest value in $X \cup Y$ gives us one of the numbers (the larger number). If all of the elements of $X \cup Y$ are factors of this number, this tells us that the smaller number is a factor of the larger number, in which case the largest value of $X \cap Y$ gives us the smaller number. If there are values in $X \cup Y$ which are *not* factors of the larger number, then the largest of these values gives us the smaller number.

In the Discussion at the end of Section D, students should find that all of the examples given *may* be rational. For example:

- $\pi + (-\pi) = 0$, which is rational.
- $(\sqrt{3}+5) - \sqrt{3} = 5$, which is rational.
- $\sqrt{2} \times \sqrt{2} = 2$, which is rational.

CHAPTER 6: LINEAR EQUATIONS AND INEQUALITIES

- **A** Linear equations
- **B** Equations with fractions
- **C** Problem solving
- **D** Linear inequalities

E Solving linear inequalities

Keywords:

- algebraic equation
- equal sign
- equate numerators
- equation
- inequality
- inequality sign
- inverse operation
- left hand side
- linear equation
- linear inequality
- lowest common denominator
- right hand side
- solution

Section A is largely revision of the work done on solving linear equations in MYP 3, and may be skipped through more quickly if students are comfortable with the material. Section B is more likely to be challenging for students, as the lowest common denominator of fractions involving variables must be found.

In Section C (Problem solving), a subsection about mixture problems has been added. Students are likely to find the process of converting the given information into an equation to be the most challenging part. Students should be encouraged to draw a diagram, as was done in the worked example, if that helps them.

One of the biggest conceptual difficulties when moving from equations to inequalities is the idea that the solution is no longer a single value of x, but an interval of infinitely many values of x. It may help students to consider a particular inequality and, by substitution, show that there are many values of x which satisfy the inequality.

When solving linear inequalities, students can use the work done in Chapter 5 to write the solutions in interval notation, and to graph the solutions on a number line.

CHAPTER 7: VENN DIAGRAMS

A Venn diagrams
B Venn diagram regions
C Numbers in regions
D Problem solving with Venn diagrams

Keywords:

- complement
- disjoint
- intersection
- number of elements
- subset
- union
- Venn diagram

This chapter presents the remainder of what was the "Sets and Venn diagrams" chapter in the previous edition. By placing linear equations between the chapters, students can use their linear equation solving skills to find unknown numbers in regions on Venn diagrams.

Section A may be worked through quickly if students are comfortable with this material from MYP 3.

Sections B (Venn diagram regions) and C (Numbers in regions) have been moved from being subsections of the Venn diagrams section, to being sections in their own right. In Section C, some more complex cases are considered than what was in MYP 3, and having linear equations behind them allows students to apply a more methodical approach to find the unknown numbers.

Apart from an extension question at the end of Section D, the problem solving work involving 3 sets has been moved to MYP 5.

CHAPTER 8: SURDS AND OTHER RADICALS

A Square roots
B Properties of radicals
C Simplest surd form
D Cube and higher roots
E Power equations
F Operations with radicals
G Division with surds

Keywords:

- cube root
- nth root
- power equation
- radical
- rationalising the denominator
- simplest surd form
- square root
- surd

This chapter is adapted from Chapter 5 (Radicals) in the previous edition.

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$.

In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations. In the Discussion at the end of Section B, students should be able to give examples such as:

- $\sqrt{16+9} = 5$, whereas $\sqrt{16} + \sqrt{9} = 7$
- $\sqrt{4-1} = \sqrt{3}$, whereas $\sqrt{4} - \sqrt{1} = 1$

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form. For example, when comparing which value is larger, it is easier to compare $\sqrt{27}$ and $\sqrt{28}$ than to compare $3\sqrt{3}$ and $2\sqrt{7}$.

Section D (Cube and higher roots) has been included in this edition. Students will see that the properties of radicals that we applied to square roots in Section B also apply to cube and higher roots.

Section E (Power equations) has also been added. Power equations are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right. We feel that it is a skill that should be explicitly taught.

We have marked the final two sections as Extended material, and should be completed by students completing the Extended content of the MYP Framework.

The sections "Adding and subtracting radicals" and "Multiplications involving radicals" in the previous edition have been combined into the single section "Operations with radicals". Since the rules for operating with radicals are identical to those for algebra, we felt there was little to be gained by keeping these sections separate.

In Section G (Division with surds), students write expressions of the form $\frac{b}{c\sqrt{a}}$ with an integer denominator by multiplying the numerator and denominator by $\sqrt{a}$. We feel that this is a sufficient introduction to division by surds for the Extended students, and have therefore removed the subsection in which the radical conjugate must be found to perform the division. This will be dealt with at MYP 5.

CHAPTER 9: PYTHAGORAS' THEOREM

A Pythagoras' theorem
B Pythagorean triples
C Problem solving

Keywords:

- hypotenuse
- Pythagoras' theorem
- Pythagorean triple
- right angled triangle

In the previous edition books, Pythagoras' theorem was studied at MYP 3 and MYP 5. In this edition, we have retained a chapter on Pythagoras' theorem in MYP 3. However, students are not required to study it, as the new MYP Framework specifies that Pythagoras' theorem should be first studied at MYP 4. We have therefore added this chapter on Pythagoras' theorem at MYP 4.

Teachers should be aware that students *may* have already seen Pythagoras' theorem at MYP 3, but given it was not required at MYP 3, this may be the first time students have encountered it.

The Opening Problem walks students through a proof establishing Pythagoras' theorem. This proof is similar to one presented in an Investigation at MYP 3, however the proof in this chapter requires the expansion of $(a-b)^2$, which the students had not seen at MYP 3.

In Section C (Problem solving), we have included some problems involving three-dimensional objects. In these problems, students must use Pythagoras' theorem twice. We do not believe these types of questions will present significant conceptual difficulties for students, and have simply presented them at the end of the section. We feel that making a separate section for them makes the problems appear harder than they actually are.

We have not included the converse of Pythagoras' theorem here, as it is marked as part of the Extended course in the MYP Framework, however we will return to it at MYP 5.

CHAPTER 10: FORMULAE

- **A** Formula construction
- **B** Substituting into formulae
- **C** Rearranging formulae
- **D** Rearrangement and substitution

Keywords:

- formula
- inverse operation
- rearrange
- subject
- substitute

In this edition, we have moved the formulae chapter before the measurement chapters, as we felt that students will benefit from doing this work before encountering the measurement formulae. As a result, we have removed any questions referring to measurement formulae, as these will be addressed in the measurement chapters. In their place, we have included more questions based on physics formulae.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 8 should help with this.

In Section C, we have removed the subsection "Formulae involving fractions on both sides", and presented all of the questions in a single section. We feel that it is not helpful to treat this type of formula as a "special case", and that these formulae can be rearranged by applying the same techniques used to rearrange the other formulae in the section.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

CHAPTER 11: FINANCIAL MATHEMATICS

- **A** Percentage increase or decrease
- **B** Business calculations
- **C** Appreciation and depreciation
- **D** Simple interest
- **E** Compound interest

Keywords:

- appreciation
- break even
- compound interest
- cost price
- depreciation
- discount
- future value
- goods and services tax
- inflation
- interest rate
- loss
- marked price
- mark-up
- multiplier
- per annum
- percentage change
- present value
- principal
- profit
- selling price
- simple interest
- value-added tax

In this chapter, students apply their knowledge of percentages in financial contexts such as discount and mark-up, appreciation and depreciation, and simple and compound interest.

The beginning of this chapter has undergone a significant restructure from the previous edition. Section A (Percentage increase or decrease) covers the material from Section A.1 in the previous edition. This is a more general consideration of percentages, and has therefore been separated from the rest of Section A from the previous edition.

Section B (Business calculations) covers what remained from Section A in the previous edition. Whereas this material was split into many subsections in the previous edition, here we have presented mark-up, discount, profit, loss, and tax in a single section, noting that each is an application of percentage change.

In Activity 1, we ask students to take the very general percentage change formula "old amount $\times$ multiplier $=$ new amount", and specify what "old amount" and "new amount" refer to in each context. For example, when considering mark-up, we have "selling price $=$ marked price $\times$ multiplier", and when considering loss, we have "selling price $=$ cost price $\times$ multiplier". We hope that this approach helps students see that each context involves the same processes, and that students should not feel that they need to memorise a distinct set of formulae for each context.

In Sections D and E, given that the students have only recently completed the Formulae chapter, we have included some questions in which students must rearrange the simple interest and compound interest formulae.

We have removed the section on personal loans in this edition, since this work involves reading values from a table, and is an unnecessary departure from the formula-based work done involving simple and compound interest. Students will solve these types of problems using the financial application of their calculator in their Diploma Programme courses. However, we have retained the Global Context project involving paying off a home loan, in which students use a spreadsheet to explore the relationship between the amount borrowed, the interest rates, the duration of the loan, and the monthly repayment.

CHAPTER 12: MEASUREMENT: LENGTH AND AREA

- **A** Units of length
- **B** Perimeter
- **C** Units of area
- **D** Area of polygons
- **E** Area of circles and sectors

Keywords:

- arc length
- area
- centimetre
- circumference
- hectare
- kilometre
- metre
- millimetre
- perimeter
- sector

In this edition, we have split the measurement chapter into two separate chapters. This chapter covers the measurement of 2-dimensional shapes, specifically length and area.

Splitting the chapters in this way allows us to provide a more complete coverage of the concepts. This is important because in this edition of the MYP books, students have done a little less measurement work at MYP 2 and MYP 3, especially with unit conversion and circles.

In this edition, we start with a section about length units and their conversions (Section A), before exploring perimeter in Section B.

Now that this chapter occurs after the Formulae chapter, we have included more questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

As with length, we have added a section about area units and their conversions. This is especially important, since in this edition of the MYP books, conversion of area units was only introduced at MYP 3, rather than MYP 2. This means that students may be less familiar with it at MYP 4.

In this edition of the MYP books, the area of a circle is not introduced until MYP 3, as opposed to MYP 2 in the previous edition. For this reason, we have divided the area formulae into two sections. Section D involves the area of polygons, all of which the students should have seen in previous years. Given that the formulae should be familiar to students, we are able to provide some more challenging questions within this section.

This allows us to provide a little more coverage to circles and sectors in Section E. We provide another demonstration to establish the area of a circle, rather than simply restating the formula. The area of a sector will be new to students, but if students understand that it is simply a fraction of the area of a circle, then it should not cause difficulties.

CHAPTER 13: MEASUREMENT: SURFACE AREA, VOLUME, AND CAPACITY

A Surface area of a solid with planar faces
B Surface area of a cylinder
C Surface area of a cone
D Surface area of a sphere
E Units of volume
F Volume of a solid of uniform cross-section
G Volume of a tapered solid
H Volume of a sphere
I Capacity

Keywords:

- apex
- capacity
- kilolitre
- litre
- megalitre
- millilitre
- net
- solid of uniform cross-section
- surface area
- tapered solid
- volume

This chapter covers the material from Sections C to E of the Measurement chapter in the previous edition. Although there now appears to be many sections, this is mainly because each formula generally has a section of its own, rather than being one of many subsections in a section.

Students should have seen the surface area work in Sections A, B, and D in MYP 3. The main difference here is the use of Pythagoras' theorem to find unknown lengths in some solids.

In Section C, students are guided through an Investigation in which the net of a cone is used to derive its surface area.

In Section D, the Investigation in the previous edition in which students discover the surface area of a sphere by comparing it to the flat surface of a hemisphere, has been moved down to MYP 3. Here, a result obtained by Archimedes is used to derive the formula for the surface area of a sphere.

Section E focuses solely on the units of volume and their conversions. As with units of area, conversion of volume units was not introduced in this edition of MYP books until MYP 3 as opposed to MYP 2, so we felt it sensible to provide a more complete coverage of volume units in this edition.

The material in Section F should be familiar to students from MYP 3. In MYP 4, a greater emphasis is placed on finding a dimension, given the volume of a solid.

In the online Activity at the end of Section F, students explore the ratio surface area : volume for prisms of various shapes. In general, students should find that, for a given volume, we can make a large surface area by making the prism extremely long and flat, whereas we can make the surface area smaller by making the prism more "regular", with sides of similar length. For example, a cube of length 1 m and a rectangular prism measuring $10 \text{ m} \times 10 \text{ m} \times 1 \text{ cm}$ each have volume 1 m^3. However, the cube has surface area 6 m^2, whereas the rectangular prism has surface area 200.4 m^2.

In Section G, students complete an Investigation to establish the volume of a square-based pyramid. Although the solids explored in the Investigation are quite specific, they should give an insight into why the formula is true for all tapered solids.

CHAPTER 14: ALGEBRA: FACTORISATION

A Common factors
B Difference between two squares factorisation
C Perfect squares factorisation
D Factorising $x^2 + bx + c$
E Miscellaneous factorisation
F Expressions with four terms
G Factorising $ax^2 + bx + c,\ a \neq 1$

Keywords:

- factorisation
- fully factorised
- quadratic trinomial
- splitting the middle term
- sum and product factorisation

In this edition of the MYP books, quadratic factorisation has been removed at MYP 3. This means that students will be seeing most of the material in this chapter for the first time. As a result, we needed to be more methodical and detailed in establishing the factorisation rules than in the previous edition, and the exercises contain more basic questions to give the students more practice.

To emphasise that factorisation is the reverse process of expansion, students should be reminded that they can check their factorisations by expanding their answer.

In Section E, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable. It may be helpful for them to realise that the difference between two squares factorisation and perfect squares factorisation are special cases of the sum and product factorisation (in difference between two squares, the required numbers p and q are negatives of each other, and in perfect squares factorisation, the numbers are the same).

In this edition, we have marked the final two sections as Extended material. Although many students will study factorisation by "splitting" the middle term in MYP 5, even those only completing the Standard course, there is already quite a lot of new material in this chapter. For the average student, it would be quite a jump to go from only seeing factorisation by removing common factors in MYP 3, to factorisation by "splitting" the middle term in MYP 4. Students looking to complete the Standard content would likely be best served by omitting the final two sections here, and then dealing with "splitting" the middle term in MYP 5.

CHAPTER 15: ALGEBRAIC FRACTIONS

A Evaluating algebraic fractions
B Simplifying algebraic fractions
C Multiplying algebraic fractions
D Dividing algebraic fractions
E Adding and subtracting algebraic fractions

Keywords:

- algebraic fraction
- evaluate
- factorise
- highest common factor
- rational expression

Placing this chapter after we have studied factorisation allows us to factorise the numerator and denominator of an algebraic fraction. This helps perform more involved simplifications than were done in Section 2G.

In the Discussion at the end of Exercise 15B.1, students can find that the simplification $\frac{2x}{x} = 2$ is incorrect when $x = 0$, since in this case the LHS is undefined.

In Section E, we have avoided using the term "simplify" as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the "simplest" in this case. Instead, we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term "simplify", and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is "better" or "simpler" may depend on what we are trying to do with the expression.

CHAPTER 16: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient
D Parallel and perpendicular lines
E Using coordinate geometry

Keywords:

- axis
- Cartesian plane
- coordinates
- distance formula
- gradient
- midpoint
- negative reciprocal
- ordered pair
- origin
- parallel lines
- perpendicular lines
- quadrant
- x-axis
- x-coordinate
- y-axis
- y-coordinate

In this edition, the material involving the equations of lines has been moved to its own chapter "Straight lines" (Chapter 17). In the content that remains, students explore the Cartesian plane, including distance between points, midpoints, and gradients. We feel this is a logical way to divide this material, since some new material has been added, and a single chapter would be very large. The work in this chapter provides students the tools to describe straight lines in the Cartesian plane in the following chapter.

While students encountered the gradient of a line in MYP 3, it was only in terms of horizontal and vertical steps. In MYP 4, students are given the gradient formula. In the Discussion in Section C, it would be useful to remind students that the gradient is a measure of the *steepness* of a line. In this sense, it is desirable that a steeper line has a higher gradient.

In the Discussion in Section D, students should find that the rules for perpendicular lines do not apply to horizontal and vertical lines, and that these are special cases which will need to be treated separately. In other words, the idea that vertical and horizontal lines are perpendicular should be intuitive, and gradient is not a very helpful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry). The questions in this section were part of Section D in the previous edition.

CHAPTER 17: STRAIGHT LINES

A Vertical and horizontal lines
B Points on a line
C Axes intercepts
D Graphing from a table of values
E Gradient-intercept form
F General form
G Finding the equation of a line

Keywords:

- axes intercepts
- equation of a line
- general form
- gradient-intercept form
- horizontal line
- point-gradient form
- table of values
- vertical line
- x-intercept
- y-intercept

This new chapter comprises the material about straight lines that was in Sections E to G of the Coordinate geometry chapter in the previous edition. Presenting this work as a chapter in its own right allows us to provide a more complete treatment of this material, without creating a chapter that is too large and encompassing too many different ideas.

Rather than leaving vertical and horizontal lines as an afterthought, they are presented in Section A in this edition. In Sections C and D, students have the opportunity to relate the y-intercept and gradient of a line with its equation. This lays the groundwork for graphing a line in gradient-intercept form in Section E. The work done in Section C is also used in Section F, where students use axes intercepts to graph lines in general form.

In Section G, we have more clearly delineated the combinations of information which can be provided in order to determine the equation of a line (gradient and y-intercept, gradient and a point, two points). Breaking these into separate subsections will allow students to focus on the nuances of individual questions within each subsection (such as whether the information is given in word form or graphical form, and whether the equation must be given in gradient-intercept or general form).

CHAPTER 18: SIMULTANEOUS EQUATIONS

- **A** Solution by trial and error
- **B** Graphical solution
- **C** Solution by equating values of y
- **D** Solution by substitution
- **E** Solution by elimination
- **F** Problem solving

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution
- trial and error

In this edition of the MYP books, simultaneous equations appears at MYP 4 and MYP 5, rather than MYP 3 and MYP 4. So, students will now be seeing simultaneous equations for the first time in MYP 4, and the chapter that appears here is adapted from the chapter that previously appeared in MYP 3.

This chapter starts by asking students to solve simultaneous equations by trial and error. It is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In the Discussion at the end of Section A, the equations in **1 a** have solution $x = -\frac{1}{6}$, $y = \frac{3}{14}$, and the equations in **1 b** have no solution. The equations in **1 c** have infinitely many solutions, so it is likely that students would have found different solutions. Trying to solve simultaneous equations by trial and error makes it hard to find non-integer solutions, and it is hard to identify when a system has no solutions or infinitely many solutions. This should lead students to conclude that a more systematic approach is required.

In Section B, a graphical approach is used. This should allow students to build on what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should better illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections C to E.

CHAPTER 19: TRANSFORMATION GEOMETRY

- **A** Translations
- **B** Reflections
- **C** Rotations
- **D** Enlargements and reductions
- **E** Stretches
- **F** Combinations of transformations

Keywords:

- centre of enlargement
- centre of rotation
- enlargement
- horizontal stretch
- image
- mirror line
- object
- reduction
- reflection
- rotation
- scale factor
- symmetry
- transformation
- translation
- vertical stretch

Students will not have seen transformations since MYP 2, so it may be worthwhile revisiting the notation and terminology associated with transformations.

In the Discussion in Section B, students should find that, if the vertices of a polygon are labelled in clockwise order, the vertices of its reflection will be labelled in anticlockwise order.

In this edition, we have removed work on line symmetry, but we have extended the work done on reflections to consider not only reflections in the axes, but also in the lines $y = x$ and $y = -x$. Reflections in the line $y = x$ will be useful in the study of inverse functions in later years.

The Discussion at the end of Section C should lead students to conclude that:

- For all of the transformations studied so far, the size and shape of the figure do not change, but the position does change.
- The orientation of a figure changes for a reflection and a rotation, but not for a translation.

In Section D, we have added an Investigation to help students understand the relationship between the centre of an enlargement, the object, and its image. This is an important conceptual jump from the enlargement work done in MYP 2, as we no longer consider the relative sizes of the object and its image, but also their positions, which is a crucial aspect of all the other transformations we have considered.

We have added the work on vertical and horizontal stretches (Section E) in this edition. This is because we will no longer have an explicit chapter on transformations at MYP 5 Extended, so the material on stretches which was previously in MYP 5 Extended has been moved here.

The way we have treated each transformation here reflects where the transformation will be used in coming years. Translations, reflections, and stretches will be used in functions, reflections and rotations will be used in trigonometry, and enlargements will be used in similarity and congruence. This leads to a natural bias in how we treat each transformation. For example, there is a bias towards work on the Cartesian plane for translations and reflections (and to a lesser extent rotations), but not enlargements.

CHAPTER 20: QUADRATIC EQUATIONS

A Quadratic equations
B Equations of the form $x^2 = k$
C The null factor law
D Solving by factorisation
E Problem solving
F Completing the square

Keywords:

- completing the square
- null factor law
- quadratic equation

Quadratic equations have been removed from MYP 3 in this edition, so this will now be the first time students have seen quadratic equations. As a result, we have moved the introductory Section A (Quadratic equations) up from MYP 3. The purpose of this section is to help students understand what a quadratic equation is, and to help students see that quadratic equations can have zero, one, or two solutions.

In Section B, students solve equations of the form $x^2 = k$. Although simple versions of these equations were solved in Section 8E (Power equations), here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In the Discussion at the start of Section C, students should conclude that if the product of two numbers is zero, then at least one of those numbers must be zero. This will lead students to the null factor law.

In Section D.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations such as $(2x+1)^2 - (x+2)^2 = 0$, it should be clear that using difference between two squares factorisation is more efficient.

In this edition, we have moved completing the square to the end of the chapter, and marked it as Extended material. This seems appropriate given this chapter is now the first time students see quadratic equations. More able students should be encouraged to attempt these quadratic equations. Almost all of the questions here involve the simplest case where $a = 1$. Completing the square will be presented again in MYP 5, including cases where $a \neq 1$.

CHAPTER 21: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Using axes intercepts to graph quadratics
F Projectile motion

Keywords:

- parabola
- projectile motion
- quadratic function
- vertex
- x-intercept
- y-intercept

We have restructured the opening of this chapter in this edition. In Section A, we define quadratic functions, and consider how to find y given x as well as x given y, before moving on to graphs of quadratic functions in Section B. We believe this is a more logical structure, and mirrors the structure of quadratic function chapters in later years, including the Diploma Programme courses.

Through an Investigation, students see how graphs of the form $y = (x - h)^2 + k$ are translations of $y = x^2$, and that graphs of the form $y = ax^2$ can be formed by stretching and reflecting the graph of $y = x^2$. Question **10** of Exercise 21C is marked in dark blue, as students who studied Section 20F (Completing the square) in the previous chapter can use completing the square to write the quadratic functions in the form $y = (x - h)^2 + k$, and can then use this form to graph the function. Because this is now Extended material, we have not dedicated a whole subsection to it, as was done in the previous edition.

In Section D, students find the axes intercepts of quadratic functions. Students should notice that some forms of the quadratic function make it easier to find the y-intercept (such as $y = x^2 + 7x - 3$), while other forms make it easier to find the x-intercepts (such as $y = 3(x + 2)(x - 4)$). Question **8** is marked in dark blue, as completing the square is required to find the x-intercepts in this question.

Section F (Projectile motion) has been added in this edition, and is largely done using technology. This is a good opportunity for students to use technology to find characteristics of the graphs such as the axes intercepts and turning points, and will be especially useful for students heading to one of the Mathematics: Applications and Interpretation courses in the Diploma Programme. Solving these problems algebraically requires completing the square with $a \neq 1$ to find the vertex, which is beyond the scope of the course at this point. The Global Context at the end of the chapter will extend students to take a more algebraic approach to the study of motion.

CHAPTER 22: CONGRUENCE AND SIMILARITY

A Congruence
B Congruent triangles
C Similarity
D Similar triangles
E Areas of similar figures
F Volumes of similar solids

Keywords:

- congruent figures
- congruent triangles
- equiangular
- same ratio
- scale factor
- similar figures
- similar triangles

Although congruence and similarity does appear in our MYP 3 book, it is not explicitly required in the MYP Framework until MYP 4. For this reason, teachers should be aware that congruence and similarity may have been left out at MYP 3, and students may be seeing it for the first time here in MYP 4.

In this edition, the "proof" questions, which were placed in Section C (Proof using congruence) in the previous edition, have been absorbed into Section B (Congruent triangles) in this edition.

The Discussion at the end of Section B should cause students to realise that, just because there is not enough information to conclude two triangles are congruent, this does not *necessarily* mean that the triangles are not congruent. For example, in the first pair of triangles, we cannot conclude the triangles are congruent, as one triangle could be an enlargement of another. However, we cannot conclude the triangles are *not* congruent, since the side lengths of the triangles may indeed be equal.

In this Discussion, it may be helpful to ask students what can be inferred by a *lack* of tick marks indicating equal side lengths. For example, in the second pair of triangles, we cannot conclude the triangles are congruent, since the equal sides are not in corresponding positions. However, if the side of the second triangle between α and β is also equal to the sides with the tick marks, then the triangles *would* be congruent. Does the lack of a tick mark mean that the side lengths are definitely different?

It might also be useful to ask to what extent we can assume the diagram is drawn reasonably to scale. For example, in the third pair of triangles, we cannot conclude that the triangles are congruent, since the equal angle is not between the equal sides. However, it can be shown (once we have knowledge of trigonometry), that if we can assume the angle at the top is acute, and the angle in the middle is obtuse (as it appears in the diagram), then there is sufficient information to conclude that the triangles are congruent. Are these reasonable assumptions to make?

Students may also enjoy critiquing Lewis Carroll's "proof" that all triangles are isosceles in the Puzzle at the end of Section B. The flaw in the proof comes in *Step 1*. The point X at which the lines meet will never occur inside the triangle. If the triangle is indeed isosceles, the two lines will be coincident, and if the triangle is not isosceles, X will lie outside the triangle.

Since this may now be the first time students are seeing congruence and similarity, we have included an introductory section about similarity (Section C). It should be highlighted that, for figures in general to be similar, both the figures must be equiangular *and* the side lengths must be in proportion. In particular, establishing one of these without the other is not sufficient. Questions **4** and **5** highlight that quadrilaterals that are equiangular may not be similar, and quadrilaterals with side lengths in the same ratio may not be similar. However, *triangles* are special in that, if one of these properties is true, the other must also be true. So, to establish two triangles are similar, only one of these properties need be proven. This is what is explored in Section D.

In Section D.2, we have added some questions which require students to solve a quadratic equation to find the unknown. This should not present a problem for students, since quadratic equations were studied in Chapter 20.

CHAPTER 23: TRIGONOMETRY

- **A** Scale diagrams in geometry
- **B** Labelling right angled triangles
- **C** The trigonometric ratios
- **D** Finding side lengths
- **E** Finding angles
- **F** Problem solving
- **G** Bearings

Keywords:

- adjacent side
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- scale diagram
- sine
- tangent
- trigonometry
- true bearing

Trigonometry has now been removed from MYP 3, since the Framework specifies that this should not be introduced until MYP 4. This means that students will now see trigonometry for the first time here.

We have therefore added Section A (Scale diagrams in geometry), which was previously in the MYP 3 trigonometry chapter. This section should be seen as making use of the similarity of a situation and its scale diagram. While these types of problems can be solved using scale diagrams, it is hard to obtain accurate answers in this manner. Trigonometry makes the process more accurate by providing the actual ratios required.

We have moved trigonometry after similarity and congruence in this edition, so students can use similarity to explain why the ratio of side lengths for triangles containing a particular angle should be equal.

When introducing the trigonometric ratios in Section C, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 24^\circ$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

This is likely to be the first time students have used their calculators for finding trigonometric ratios, so if their calculators are not producing the expected answers, it may be because their calculators are not set to "degree" mode. If needed, the graphics calculator instruction icon can help with this.

In the Discussion at the end of Section C, students should find that $\sin\theta$ and $\cos\theta$ can take values between 0 and 1 (a useful first step here is to recognise that the hypotenuse is the longest side of the triangle), and that $\tan\theta$ can take any positive value.

In the Puzzle at the end of Section E, students should find that 16 triangles can be drawn before they start to overlap.

In this edition, we have removed Section G (3-dimensional problem solving). This seemed reasonable given that this is now the first time students have studied trigonometry. These problems will be studied in MYP 5.

CHAPTER 24: DEDUCTIVE GEOMETRY

- **A** Deductive geometry
- **B** Midpoint theorem
- **C** Angle in a semi-circle theorem
- **D** Chords of a circle theorem
- **E** Radius-tangent theorem
- **F** Tangents from an external point theorem

Keywords:

- chord
- deductive geometry
- midpoint
- radius
- semi-circle
- tangent

This chapter brings together a lot of what students have learnt so far in the book about Pythagoras' theorem, congruence and similiarity, and trigonometry.

In this edition, we have moved the midpoint theorem to Section B, so that we present all of the non-circle work first.

We have also chosen to present each theorem in a separate section, rather than having a large "Circle theorems" section. This allows us to more logically start each section with a proof of the theorem (where appropriate) before applying the theorem.

We have removed the blue box which describes the terminology associated with circles (such as chord, diameter, tangent), as students should be familiar with these terms from previous years.

If students are having trouble with any of the circle geometry theorems, keep in mind that students will have another opportunity to see these theorems in MYP 5, as well as being introduced to some new circle theorems.

CHAPTER 25: PROPORTION

- **A** Direct proportion
- **B** Powers in direct proportion
- **C** Inverse proportion
- **D** Powers in inverse proportion

Keywords:

- directly proportional
- hyperbola
- inversely proportional
- proportionality constant

In Section A, the problem solving questions now flow directly from the introductory work on direct proportion, rather than being placed in a separate subsection.

In Example 2 of Section A, two methods are presented to find the unknown value. Students should be encouraged to see that Method 2 is the preferred approach, as it avoids the need to determine the proportionality constant k and the law connecting y and n, and instead uses the properties of the proportionality relationship. The difference in methods becomes even more pronounced as we deal with powers and with inverse proportion, and Method 2 is used in the subsequent worked examples in the chapter.

In the Discussion at the end of Section A, students should realise that there are many variables in their everyday life that are in direct proportion. For example, *distance* and *speed* are directly proportional, so if they travel twice as fast, they will travel twice the distance in a given time period.

In the Discussion at the end of Section B, students should find that the *volume* and the *mass* of the models are directly proportional to the *cube* of the *height*, while the *surface area* is directly proportional to the *square* of the height. However, we would not expect the measurements to match exactly with these relationships, since there will always be imperfections and inconsistencies with how they are made, and our measurements are likely to contain some error.

If the models are made with different materials, we would still expect the *volume* and *surface area* to have the same proportional relationships with *height* (as these do not depend on the material used). However, it is likely that there is no longer a proportional relationship between *mass* and *height*.

In the Research activity at the end of the chapter, students must research how the rate in which a fire spreads is affected by various factors. Students should remember that, while there may be a relationship between the variables listed (for example, if one increases, the other decreases), this does not necessarily imply a proportionality relationship. Before deciding whether a proportionality relationship exists, students must first decide how they will *measure* some of the variables. For example, gradient could be used as a measure of the steepness of the land. Students could discuss whether different choices for the measurement of the variable could affect the nature of the proportionality relationship.

CHAPTER 26: PROBABILITY

- **A** Sample space and events
- **B** Theoretical probability
- **C** Probabilities from Venn diagrams
- **D** Independent events
- **E** Dependent events
- **F** Probabilities from tree diagrams
- **G** Experimental probability
- **H** Probabilities from tabled data
- **I** Expectation

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complement
- compound events
- dependent events
- event
- expectation
- experimental probability
- frequency
- impossible event
- independent events
- probability
- relative frequency
- sample space
- theoretical probability
- tree diagram
- Venn diagram

The order in which the material is presented has been changed for this edition. Experimental probability has been moved from the start of the chapter to the end, since the ideas behind finding experimental probabilities are grounded in the work done in calculating theoretical probabilities.

Section A has been restructured to not only consider the different ways to represent the sample space of an experiment, but also to define an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, and also means the idea of an event has already been introduced when we come to calculating theoretical probabilities in Section B.

Another advantage of this approach is that it allows us to talk more generally about finding theoretical probabilities in Section B, without having to deal with 2-dimensional grids in a separate section. This is because we have already discussed how to identify an event's outcomes on a grid in Section A.

Finding probabilities from Venn diagrams has been moved forward in the chapter to Section C. This is because we only deal with single events, so it makes sense to deal with it before we move on to compound events. The method for finding unknown numbers in regions has been updated to reflect the more formal method described in Chapter 7. The only difference here is that, once the numbers of elements in the relevant regions have been found, we divide through by $n(U)$ to find the corresponding probability.

Section H from the previous edition (Sampling with and without replacement) has been absorbed into the Probabilities from tree diagrams section, since all of this work is done using tree diagrams anyway, and there is little value in placing them in separate sections.

In the Global Context on genetics at the end of the chapter, Punnet squares are used to calculate the probabilities of particular inherited traits occurring. Students should find this to be an interesting application of the material they have studied in this chapter.

CHAPTER 27: STATISTICS

- **A** Types of data
- **B** Discrete numerical data
- **C** Continuous numerical data
- **D** Describing the distribution of data
- **E** Measures of centre

- **F** Cumulative frequency graphs
- **G** Measures of spread
- **H** Box plots
- **I** Comparing numerical data

Keywords:

- back-to-back bar chart
- back-to-back histogram
- back-to-back stem-and-leaf plot
- biased sample
- bimodal
- bimodal distribution
- box plot
- categorical data
- categorical variable
- census
- class interval
- column graph
- continuous numerical variable
- cumulative frequency
- cumulative frequency graph
- data
- discrete numerical variable
- distribution
- dot plot
- five-number summary
- frequency histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value
- modal class
- mode
- negatively skewed distribution
- numerical data
- numerical variable
- outlier
- parallel box plot
- population
- positively skewed distribution
- range
- sample
- scale
- side-by-side column graph
- statistics
- stem-and-leaf plot
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

The "Discrete numerical data" section has been split into three: ungrouped discrete data, grouped discrete data, and stem-and-leaf plots.

Describing the distribution of a data set is new for this year, so we have decided to put this material in its own section. It is also now introduced *after* continuous numerical data, because we feel that it makes more sense to talk about the shape of data after we have covered all the data types. Outliers in the context of graphs are also introduced here.

The Discussion at the start of Section E deals with the use of the word "bimodal" in terms of frequency versus shape. A data set described as having a "bimodal distribution" may not necessarily have two modes. However, what we mean by "bimodal" should be clear from the context and how the question is worded.

Investigation 1 on page 473 is an opportunity for students to see how changing the sample size affects the accuracy of estimates of measures of centre.

The Comparing numerical data section is new for this edition. Here we introduce some graph types which may be unfamiliar to students, such as the back-to-back histogram and back-to-back stem-and-leaf plot. These should be extensions of the graphs students have encountered before. In particular, we introduce the parallel box plot which we cover in more detail in the MYP 5 books. The exercise questions in this section are a good opportunity for students to practise the concepts learnt throughout this chapter.

Investigation 2 at the end of Section I is an opportunity for students to explore a much larger data set in a spreadsheet. Rather than counting data values and doing calculations by hand, we feel that this Investigation is best done using the functions provided in the spreadsheet software used to open the data. The "Pivot table" sheet may not work in spreadsheet software other than Microsoft Excel. However, similar functionality should be available in other spreadsheet software.

CHAPTER 28: NETWORKS

- **A** Networks
- **B** Routes on networks
- **C** Shortest route problems

D Eulerian networks

Keywords:

- adjacent vertices
- arc
- circuit
- connected network
- cycle
- degree
- directed graph
- disconnected network
- edge
- Eulerian circuit
- Eulerian network
- Eulerian trail
- graph
- network
- node
- path
- semi-Eulerian network
- shortest route problem
- trail
- vertex
- walk
- weighted network

In the previous edition of our MYP books, networks were covered in an online chapter at MYP 3. In the most recent update, the MYP Framework has placed networks as Extended material for MYP 4-5. We have chosen to put this material at MYP 4 and mark it as Extended work, since there is already so much content to cover at MYP 5.

One of the most challenging aspects for students starting their study of networks is the amount of terminology involved. Section A is designed to allow students to become familiar with the concept of a network, as well as the terminology used. Students should realise that a network need not refer to a physical connection between objects. For example, some networks in Exercise 28A describe friendships between people, or matches played in a tournament.

In Section B, students use terminology to describe routes between vertices. The terminology used here is the same as that used in the Mathematics: Applications and Interpretation HL course in the Diploma Programme.

In Section C, students investigate the shortest path between vertices on a weighted network. This is first done by trial and error, but students should see that this approach is impractical as the network becomes larger. In Section C.2, Dijkstra's algorithm is used to find the shortest path in a network.

Section D (Eulerian networks) is not explicitly mentioned in the Framework, but it is an interesting application of networks. Also, Eulerian networks feature quite heavily in the graph theory component of the Mathematics: Applications and Interpretation HL course in the Diploma Programme, so students planning to complete this course will benefit from an introduction to Eulerian networks here.

In the Discussion at the end of Section D, students should discover that *every* network contains an even number of vertices of odd degree. To explain this, students should first think about why the total sum of the degrees of the vertices in a network must be even. Then, they should think whether this would be possible if there were an *odd* number of vertices of odd degree.

The Global Context at the end of the chapter explores the relationship between the number of vertices, faces, and edges of a planar network. Students can also see how this relationship can be applied to three-dimensional polyhedra, and can use this relationship to prove some interesting properties of polyhedra.

CHAPTER 29: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

A Trigonometry with obtuse angles
B The area of a triangle
C The sine rule
D The cosine rule
E Problem solving

Keywords:

- cosine rule
- included angle
- sine rule
- unit circle

Non-right angled triangle trigonometry is only mentioned as part of the Extended course in the MYP Framework, and for this reason we have marked the material as Extended, and placed it online. However, we would strongly encourage classes to complete this chapter if time permits, as it will be beneficial for students to be introduced to non-right angled trigonometry at this time. The concept of extending the definition of trigonometric ratios beyond acute angles is likely to be quite challenging for students, and if students can at least be introduced to this concept now, it will help them for their future study in trigonometric functions and identities.

In Section A, we define the trigonometric ratios for obtuse angles. It may be beneficial to students to see that, although it is not meaningful to talk about the trigonometric ratios of obtuse angles based on our original definition involving right angled triangles, we can extend our definition of trigonometric ratios in a way that is consistent with our original definition, but is also meaningful for a greater range of angles. This is not dissimilar to what we did in Chapter 3, when we extended the definition of exponents so that it is meaningful for zero and negative exponents.

In Section A we also establish the relationship between the trigonometric ratios of supplementary angles, which will be useful later in the chapter.

In Section B (The area of a triangle), Question **2** should help students see that the standard formula for the area of a triangle $A = \frac{1}{2} \times \text{base} \times \text{height}$, is a special case of the formula given here, in which the included angle is a right angle.

In Section C.2, students use the sine rule to find unknown angles. At MYP 4, we avoid the need for students to consider the ambiguous case by choosing to draw the diagrams approximately to scale. Students can therefore use the diagram to determine whether the unknown angle is acute or obtuse. The ambiguous case of the sine rule will be considered more completely in MYP 5.

In Section D (The cosine rule), students may benefit from seeing that Pythagoras' theorem is a special case of the cosine rule, in which the included angle is a right angle.

CHAPTER 30: MATHEMATICAL LOGIC

A Propositions
B Compound propositions
C Implication
D Equivalence
E Constructing truth tables
F Tautology and logical contradiction
G Logical equivalence

Keywords:

- compound proposition
- conjunction
- disjunction
- equivalence
- implication
- logical contradiction
- logically equivalent
- negation
- proposition
- tautology
- truth table
- truth value

This online chapter is marked as Extended material as it is not part of the MYP Framework. It is mainly here for students intending to study the Mathematics: Analysis and Approaches HL course in the Diploma Programme, as it is a useful introduction to the ideas surrounding mathematical proof.

When working through this material, students should be reminded that the way that many of the terms in mathematical logic are used, differ slightly from how the terms are used in everyday life. For example, the idea that a false proposition implying another false proposition takes the truth value "true" is likely to be challenging for many students.

When we use an "if A, then B" statement in everyday usage, there is an inherent understanding that the propositions A and B are meaningfully related and dependent on each other. Mathematical logic, however, makes no judgement about the relationship between the propositions A and B, it simply looks at the truth values of each, and returns the truth value of the implication $A \Rightarrow B$. So, statements that we would not consider to be "true" in everyday usage, may be given the truth value "true" in mathematical logic.

In mathematical logic, it is perfectly acceptable for a statement such as the implication "$x > 10 \Rightarrow x > 5$" to be true for some values of x, and false for other values of x. However, when we think of such statements being "true" in everyday usage, what we generally mean is that it is true for *all* values of x.