MATHEMATICS 10 MYP 5 (Standard) third edition

Chapter summaries

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ASSUMED KNOWLEDGE 1: NUMBER

A Exponent notation
B Order of operations
C Absolute value
D Rounding numbers
E Estimation
F Percentage
G Percentage change

Keywords:

- absolute value
- base
- BEDMAS
- decimal place
- exponent
- exponent notation
- index
- multiplier
- one figure approximation
- percentage
- place value
- power
- round
- significant figure

This online chapter contains some revision of the basics of number. The material on negative numbers, fractions, and decimals from this chapter in the previous edition has been removed, as we feel that students have seen this material for many years.

Students who need this help should be encouraged to study the chapter out of classroom time, since most students will not need this chapter, and classroom time spent on this chapter is likely to come at the expense of time spent on the more advanced chapters at the end of the year.

ASSUMED KNOWLEDGE 2: ALGEBRA

A Algebraic notation
B Writing expressions
C Algebraic substitution
D The language of algebra
E Collecting like terms
F Algebraic products

Keywords:

- coefficient
- constant
- equation
- exponent notation
- expression
- like terms
- product
- product notation
- substitution
- term
- variable

This online chapter has been included for students who feel they need extra help with the basics of algebra. It is essentially revision of the work done in Chapter 2 (Algebra: Expressions) of the MYP 4 book. As with Assumed Knowledge 1, this chapter should be done out of classroom time if possible.

CHAPTER 1: EXPONENTS

A Exponent laws
B Standard form (scientific notation)

Keywords:

- base
- exponent
- exponent laws
- exponent notation
- index
- power
- scientific notation
- standard form

In this edition, "indices" have been renamed "exponents" to match the terminology used in the MYP Framework.

It is assumed that students will be familiar with exponent notation at this point, and Section A moves straight into the exponent laws. Students who need more practice with exponent notation can access some revision work in Assumed Knowledge 1 Section A.

In Section B, we have adjusted our approach to explaining standard form, to give students more guidance in how to choose their values of a and k in $a \times 10^k$. This section extends what was done in MYP 4 by also considering the addition and subtraction of numbers in standard from. Students should write the terms with the same power of 10 to perform the addition or subtraction. Through practice, students should see that it is better to adjust the smaller value so that its power of 10 matches that of the larger value. This is because the power of 10 of the larger value is more likely to be the correct power of 10 required to write the final answer in standard form.

CHAPTER 2: ALGEBRA: EXPANSION

A The distributive law
B The product $(a+b)(c+d)$
C The difference between two squares
D The perfect squares expansion
E Further expansion
F The binomial expansion

Keywords:

- binomial
- binomial expansion
- difference between two squares
- distributive law
- expansion
- factorisation
- FOIL rule
- perfect squares expansion

In this edition, we have split algebraic expansion and factorisation into two separate chapters. This gives us the opportunity to provide a more complete coverage of algebraic expansion than what was given in the previous edition. Given that students will have only seen most of this material for the first time in MYP 4, it seems reasonable to spend a little more time at MYP 5 to reinforce this work.

In Section C, some of the expansions involve working with surds. Although this comes before Chapter 7 (Surds and other radicals), it only involves very intuitive calculations such as $(\sqrt{2})^2 = 2$ based on the definition of a surd, which students should be familiar with from previous years.

In Section E (Further expansion), it may help students to recognise that they have already encountered the idea of "multiplying each term in the first bracket by each term in the second bracket", since that is what happens when applying the FOIL rule to $(a+b)(c+d)$ in Section B.

Section F (The binomial expansion) has been added in this edition. It is not required by the MYP Framework, so it has been marked in dark blue. Students looking to study the Mathematics: Analysis and Approaches SL course, or either of the HL courses in the Diploma Programme would benefit from being introduced to the binomial expansion at MYP 5. The Investigation on Pascal's triangle at the end of the section gives students a hint towards generating a general binomial expansion, which students may explore further in later years.

CHAPTER 3: ALGEBRA: FACTORISATION

A Common factors
B Difference between two squares factorisation
C Perfect squares factorisation
D Expressions with four terms
E Factorising $x^2 + bx + c$
F Miscellaneous factorisation
G Factorising $ax^2 + bx + c,\ a \neq 1$

Keywords:

- difference between two squares
- factorisation
- fully factorised
- linear factors
- perfect squares
- quadratic trinomial
- splitting the middle term
- sum and product method

In this edition, algebraic factorisation has been separated from expansion into its own chapter. As with expansion, this will allow us to provide a more complete coverage of the factorisation material.

To emphasise that factorisation is the reverse process of expansion, students should be reminded that they can check their factorisations by expanding their answer.

Students should be familiar with the material in Sections A to C from MYP 4, so these sections can be worked through quickly if needed.

In the Discussion in Section E, students should find that:

- If the sum and product of two numbers are both positive, the numbers must both be positive.
- If the sum and product of two numbers are both negative, the numbers are opposite in sign, and the negative number has the largest absolute value.
- If the sum is positive and the product is negative, the numbers are opposite in sign, and the positive number has the largest absolute value.
- If the sum is negative and the product is positive, the numbers must both be negative.

In Section F, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable.

In Section G, students will study factorisation by "splitting" the middle term. Students looking to study the Mathematics: Analysis and Approaches SL course in the Diploma Programme must complete this section, since knowledge of this factorisation method will be assumed in that course. Students looking to study the Mathematics: Applications and Interpretation SL course may choose not to complete this section, since it will not be required in that course. Students who are unsure which course they will study in the Diploma Programme should be encouraged to complete the section.

It may help students to see that this is a more general approach to the method used in Section E, since when $a = 1$ the method is essentially reduced to the sum and product method. The only difference is that the factorisation must be completed using the technique studied in Section D. Students should convince themselves that the order in which they write the split terms does not matter, since the resulting factorisation will be the same in either case.

CHAPTER 4: SETS

A Sets
B Complement of a set
C Intersection and union
D Special number sets
E Interval notation

Keywords:

- complement
- complementary sets
- disjoint
- element
- empty set
- equal sets
- finite set
- infinite set
- integers
- intersection
- interval notation
- irrational numbers
- member
- mutually exclusive
- natural numbers
- negative integers
- positive integers
- rational numbers
- real numbers
- set
- subset
- union
- universal set

In this edition, we have split Sets and Venn diagrams into two chapters, with "Linear equations and inequalities" between them. This way, interval notation can be introduced in Sets before it is used in linear inequalities, and linear equations can be used to solve problems involving Venn diagrams.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

We have endeavoured to extend what was done in the MYP 4 sets chapter by giving more opportunities to explore the concepts of finite and infinite sets. In the Discussion at the end of Section D, students should consider that, when we have two *finite* sets A and B where A is a subset of B, it is clear that there are more elements in B than in A. However, it is less clear when A and B are both infinite sets! It is tempting to say that there are about twice as many elements in $\mathbb{Z}$ as in $\mathbb{Z}^+$, since $\mathbb{Z}$ contains all the elements of $\mathbb{Z}^+$, as well as the corresponding negative integers. But does it even make sense to say that one infinite set has *more* elements than another infinite set? A potentially more illuminating example may be if A is the set of even integers, and B is the set of positive integers. Clearly A is a subset of B, but one could easily generate each element of A by multiplying each element of B by 2! Does this mean that, in some sense, they have the same number of elements?

A similar question occurs in the second dot point. Since the interval of numbers from 0 to 1 appears smaller than the interval of numbers greater than 1, it seems logical that there are more numbers greater than 1 than there are numbers between 0 and 1. However, for each number greater than 1, there is a corresponding number between 0 and 1, which is found by taking the reciprocal of the original number. Does this correspondence mean that there are the same number of real numbers between 0 and 1 as there are greater than 1?

When dealing with these questions, students should be reminded that rules that apply to finite sets do not necessarily extend to infinite sets.

CHAPTER 5: LINEAR EQUATIONS AND INEQUALITIES

A Linear equations
B Problem solving with equations
C Linear inequalities
D Problem solving with inequalities

Keywords:

- algebraic equation
- double inequality
- equal sign
- equation
- inequality
- inverse operations
- left hand side
- linear equation
- linear inequality
- right hand side

In this edition, we have removed the section on maintaining balance of equations, as students are likely to have seen this since MYP 2.

Section A is largely revision of the work done on solving linear equations in previous years, and may be skipped through more quickly if students are comfortable with the material.

In Section B (Problem solving with equations), we have included a question involving converting recurring decimals to rational numbers. This was in Section 2B (Special number sets) in the previous edition, but it seems more logical to place it in this chapter, since it requires solving a linear equation.

In this edition, we have removed the Inequalities chapter, since linear programming is no longer part of the course. We have therefore moved linear inequalities to this chapter, alongside linear equations. When solving linear inequalities, students can use the work done in Chapter 4 to write the solutions in interval notation, and to graph the solution on a number line.

In this edition, we include double inequalities, such as $5 < 3x - 4 < 17$. When solving double inequalities, students must remember to write the expressions in the solution from smallest to largest. So, if students have obtained the solution $8 > x > 5$, they must write this as $5 < x < 8$. Students should not think of this as a mathematical step of "reversing the inequality signs" in the same way as when they multiply or divide by a negative number. Instead, it is more about rewriting the solution in the correct format.

In the Discussion at the end of Section C, students should find that double inequalities such as $x + 1 \leqslant 3x - 5 < 10$ can be solved by treating them as two separate inequalities, and then combining the solutions. For example, the solution to $x + 1 \leqslant 3x - 5$ is $x \geqslant 3$, and the solution to $3x - 5 < 10$ is $x < 5$. So, the solution to the double inequality occurs where *both* of these are true, which is the interval $3 \leqslant x < 5$.

CHAPTER 6: VENN DIAGRAMS

A Venn diagrams
B Venn diagram regions
C Numbers in regions
D Problem solving with Venn diagrams

Keywords:

- complement
- disjoint
- element
- intersection
- mutually exclusive
- set identity
- subset
- union
- universal set
- Venn diagram

In this edition, Venn diagrams have been separated from the Sets chapter, and placed after linear equations, so that students can use their linear equation solving skills to find unknown numbers in regions on Venn diagrams.

In MYP 5, the work is extended to consider more Venn diagrams with three sets, and to prove some set identities.

In Section D, the problems involving 3 sets may appear challenging, but the combination of information given is such that the number of elements in all three sets can be found immediately, and the rest of the regions can be deduced quite easily from the remaining information.

In the Puzzle at the end of Section D, students should be encouraged to place numbers in the Venn diagram, starting with 1, 2, 3, ... and so on, until each region contains at least one element. The smallest element in each region then forms the universal set.

The material in Section 2H (The algebra of sets) in the previous edition has been removed, as it is not part of the MYP Framework.

CHAPTER 7: SURDS AND OTHER RADICALS

A Radicals
B Properties of radicals
C Simplest surd form
D Power equations
E Operations with radicals
F Division with surds

Keywords:

- cube root
- integer denominator
- nth root
- power equation
- radical
- rationalising the denominator
- simplest surd form
- square root
- surd

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$. In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations.

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form. For example, when comparing which value is larger, it is easier to compare $\sqrt{27}$ and $\sqrt{28}$ than to compare $3\sqrt{3}$ and $2\sqrt{7}$.

In this edition we have added Section D (Power equations). We feel that it is important to consider power equations, as they are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right.

In the Discussion at the end of Section D, students should find that, when both sides of an equation are squared, we often introduce an extra solution. There are often good reasons to square both sides of an equation, but we need to be aware that this may create extra solutions, and check whether the solutions we obtain all satisfy the original equation.

The sections "Adding and subtracting radicals" and "Multiplications involving radicals" in the previous edition have been combined into the single section "Operations with radicals". Since the rules for operating with radicals are identical to those for algebra, we felt there was little to be gained by keeping these sections separate.

In Section F (Division with surds), students write expressions of the form $\frac{b}{c\sqrt{a}}$ with an integer denominator by multiplying the numerator and denominator by $\sqrt{a}$. We have removed the material in which the radical conjugate must be found to perform the division, but have placed an Investigation about the radical conjugate in its place. Students should be encouraged to complete this Investigation, since division by surds using the radical conjugate is covered in all of the Diploma Programme courses.

CHAPTER 8: PYTHAGORAS' THEOREM

A Pythagoras' theorem
B Pythagorean triples
C Problem solving
D The converse of Pythagoras' theorem

Keywords:

- converse of Pythagoras' theorem
- Pythagoras' theorem
- Pythagorean triple

The material in Section 6A (Solving $x^2 = k$) in the previous edition has been moved to Section D (Power equations) of Chapter 7 (Surds and other radicals). This allows us to focus on Pythagoras' theorem in this chapter.

In this edition, we moved the converse of Pythagoras' theorem to the end. This allows us to consider the non-contextual and contextual applications of Pythagoras' theorem first, before considering the converse.

The section on circle problems has been removed in this edition, and these problems have been moved to the appropriate sections of Chapter 16 (Circle geometry). This approach means that we do not need to present the relevant circle theorems twice.

The questions involving three-dimensional objects have been absorbed into Section C (Problem solving), rather than being presented as a section of their own. We feel that presenting them in a section of their own makes them seem more difficult than they really are. Similarly, we felt there was no need to have a specific section about navigational problems, and have absorbed the relevant questions into the problem solving section.

CHAPTER 9: ALGEBRAIC FRACTIONS

A Evaluating algebraic fractions
B Simplifying algebraic fractions
C Multiplying algebraic fractions
D Dividing algebraic fractions

E Adding and subtracting algebraic fractions
F Equations with algebraic fractions

Keywords:

- algebraic fraction
- equating numerators
- evaluate
- lowest common denominator
- lowest terms
- rational expression
- reciprocal

The structure of this chapter remains largely unchanged from what occurred in the previous edition. The main difference is that multiplication and division of algebraic fractions have been split into separate sections.

Placing this chapter after we have studied factorisation allows students to factorise the numerator and denominator of an algebraic fraction. This helps in cancelling common factors when simplifying, multiplying, or dividing algebraic fractions.

In Section E, we have avoided using the term "simplify" as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the "simplest" in this case. Instead we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term "simplify", and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is "better" or "simpler" may depend on what we are trying to do with the expression.

In Section F, we have greatly expanded the types of equations involving algebraic fractions which can be solved, in particular to make use of the addition and subtraction of algebraic fractions done in Section E. We have also included some problem solving questions.

CHAPTER 10: FORMULAE

A Formula construction
B Substituting into formulae
C Rearranging formulae
D Rearrangement and substitution
E Predicting formulae

Keywords:

- formula
- inverse operations
- rearrange
- subject
- substitution

In this edition, we have moved the Formulae chapter before the Measurement chapter, as we felt that students will benefit from doing this work before encountering the measurement formulae. As a result, we have removed any questions referring to measurement formulae, as these will be addressed in the Measurement chapter. In their place, we have included more questions based on physics formulae.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 7 should help with this.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

In Section E, students observe the first few numbers in a sequence, identify the pattern in the sequence, and use this information to generate a formula for the nth term.

CHAPTER 11: MEASUREMENT

A Length and perimeter
B Area
C Surface area
D Volume

E Capacity

Keywords:

- arc length
- area
- capacity
- centimetre
- circumference
- hectare
- kilometre
- length
- metre
- millimetre
- perimeter
- surface area
- tapered solid
- volume

Now that this chapter occurs after the formulae chapter, we have included more questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

In Section A, we have removed perimeter formulae for specific polygons on the basis that it is more important to understand what the perimeter is, at which point the formulae are not helpful.

The Global Context at the end of Section A provides students an opportunity to explore the Minard map. The map displays six different data sets for the march of Napolean's Grand Army into Russia in 1812. As well as bringing together many aspects of mathematics, students with an interest in history are likely to enjoy this activity.

Since conversion of area and volume units were introduced later in this edition of MYP books than the previous edition, we have given students some more practice at these conversions in this edition.

It is important that classes move through this content quickly if students are comfortable with the material, as there is a lot of other more rigorous content which must be covered in MYP 5.

CHAPTER 12: QUADRATIC EQUATIONS

A Equations of the form $x^2 = k$
B The null factor law
C Solving by factorisation
D Completing the square
E The quadratic formula
F Problem solving

Keywords:

- completed square form
- completing the square
- discriminant
- null factor law
- quadratic equation
- quadratic formula

In Section A (Equations of the form $x^2 = k$), students may notice that simple versions of these equations were solved in Section 7D (Power equations). Here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In this edition, we have given the null factor law a section of its own (Section B). We feel this is appropriate seeing as quadratic equations have now been removed from MYP 3, so this is now only the second time students have encountered quadratic equations.

For the same reason, we have included more practice questions for solving quadratics by factorisation, and have split the section into subsections to separate the different types of factorisation required.

In Section C.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations, using difference between two squares factorisation is more efficient.

In Section C.3, questions that require factorisation by "splitting" the middle term are marked in dark blue. Only students who completed Section 3G (Factorising $ax^2 + bx + c$, $a \neq 1$) should complete these questions.

Sections D (Completing the square) and E (The quadratic formula) are also marked dark blue, as they are not required by the MYP Framework. However, we would encourage students to complete these sections, especially students heading towards the Mathematics: Analysis and Approaches SL course or one of the HL courses in the Diploma Programme.

In Section F, any questions requiring factorisation by "splitting" the middle term, completing the square, or using the quadratic formula, have been marked in dark blue.

In this edition, similarity has been moved later in the book, and now appears after quadratic equations. For this reason, the questions involving similar triangles have been removed from Section F, and now appear in the Congruence and similarity chapter.

CHAPTER 13: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient
D Parallel and perpendicular lines
E Using coordinate geometry
F The equation of a line
G Graphing straight lines
H Finding the equation of a line
I Perpendicular bisectors
J 3-dimensional coordinate geometry

Keywords:

- axes intercepts
- Cartesian plane
- coordinates
- distance formula
- equation of a line
- general form
- gradient
- gradient formula
- gradient-intercept form
- midpoint
- negative reciprocals
- number plane
- ordered pair
- origin
- parallel lines
- perpendicular bisector
- perpendicular lines
- point-gradient form
- quadrant
- x-axis
- X-axis
- x-coordinate
- x-intercept
- y-axis
- Y-axis
- y-coordinate
- y-intercept
- Z-axis

In this edition, we have moved this chapter later in the book, so that it occurs after quadratic equations. This allows us to use quadratic equations to solve problems involving the distance between points in Section A.

In the Discussion in Section D, students should find that the rule for gradients of perpendicular lines does not apply to horizontal and vertical lines, since vertical lines have undefined gradient. The idea that vertical and horizontal lines are perpendicular should be intuitive to students, and gradient is not a very useful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry). Many of the questions in this section were part of Section D in the previous edition.

Section F (The equation of a line) is largely a new section for this edition, in which students convert between different forms of the equation of a line, and decide whether points lie on given lines. The section of the same name in the previous edition was largely concerned with *finding* the equation of a line given particular information. This work now appears in Section H.

Section G (Graphing straight lines) is also a new section for this edition. We felt that graphing a straight line from its equation is an important skill. It will be used when solving simultaneous equations in the following chapter.

We have restructured the text in Section H so that, rather than focusing on the form in which to write the equation, we consider the different combinations of information students may be given about a line, and how to find the equation of the line in each case.

Section I (Perpendicular bisectors) is a useful application of the work students have been studying. It will also be useful for students planning on completing one of the Mathematics: Applications and Interpretation courses in the Diploma Programme, since Voronoi diagrams, which involve perpendicular bisectors, are part of these courses.

In Section J (3-dimensional coordinate geometry), students should be reminded that, although it is harder to visualise coordinates in three dimensions, most of the work that is done (such as finding distances and midpoints) extends fairly logically from what they have seen in two dimensions.

CHAPTER 14: SIMULTANEOUS EQUATIONS

A Graphical solution
B Solution by substitution
C Solution by elimination
D Problem solving
E Non-linear simultaneous equations

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution

When studying simultaneous equations, it is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In Section A, a graphical approach is used. This should allow students to use what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections B and C.

Section E (Non-linear simultaneous equations) is not part of the MYP Framework, and has been marked in dark blue. Some of these problems will require factorisation by "splitting" the middle term, or the quadratic formula.

CHAPTER 15: CONGRUENCE AND SIMILARITY

A Congruent triangles
B Proof using congruence
C Similar triangles
D Areas and volumes of similar objects

Keywords:

- congruent figures
- congruent triangles
- equiangular
- similar figures
- similar triangles

Since students will have studied congruence in MYP 3 and MYP 4, in this edition we have decided to remove the introductory section about congruent figures, and instead move straight into congruence of triangles.

Students may find the Investigation at the end of Section B to be quite subtle. For example, in **2**, students should be reminded that they cannot assume that the sides of the triangle are equal, since that is what we are trying to prove. In part **a**, we can only establish that two pairs of sides are equal, and the equal angles are not between the equal sides. So, we cannot conclude that the triangles formed are congruent. In part **b**, we can use AAcorS to prove that the triangles are congruent, and hence the large triangle is isosceles.

The Investigation on the midpoint theorem in the previous edition has been removed, and most of the material has been moved to the Deductive geometry chapter of MYP 4.

As with congruence, we have decided to remove the introductory section about similarity, and move straight into similar triangles (Section C).

In Section C, some questions which involve solving a quadratic equation have been added. These questions were in the quadratic equations chapter in the previous edition, but needed to be moved here because the quadratic equations chapter is now placed before similarity. Questions requiring factorisation by "splitting" the middle term or the quadratic formula have been marked in dark blue.

Whereas areas and volumes of similar objects were given separate sections in MYP 4, they are considered in one section here. This gives more opportunity to explore the relationships between lengths, surface areas, and volumes of similar three-dimensional objects.

CHAPTER 16: CIRCLE GEOMETRY

A Angle in a semi-circle theorem
B Chords of a circle theorem
C Radius-tangent theorem
D Tangents from an external point theorem
E Angle between a tangent and a chord theorem
F Angle at the centre theorem
G Angles subtended by the same arc theorem
H Cyclic quadrilaterals
I Tests for cyclic quadrilaterals

Keywords:

- chord
- concyclic
- cyclic quadrilateral
- diameter
- inscribed
- radius
- semi-circle
- subtended
- tangent

In this edition, rather than dividing the circle theorems arbitrarily into a "Circle theorems" section and a "Further circle theorems" section, each theorem is given a separate section.

Since the "Circle problems" section of the chapter on Pythagoras' theorem has been removed, some of the questions that were in that section have been added to this chapter.

In this edition, we have removed the "Geometric proof" section. The problems in this section have been moved to the section corresponding to the relevant circle theorem. An advantage with this approach is that students can prove each circle theorem at the time that the theorem is used, rather than leaving all of the circle theorem proofs until the end.

Some of the problems that involved trigonometry have been removed in this edition. This is because the Circle geometry chapter has been moved earlier in the book, and is now placed before trigonometry.

In this edition, we have split "Cyclic quadrilaterals" (Section H) and "Tests for cyclic quadrilaterals" (Section I) into separate sections. We have marked them as extension work as they are not mentioned in the MYP Framework, however we would encourage classes to complete these sections if time permits.

CHAPTER 17: TRIGONOMETRY

A Labelling right angled triangles
B The trigonometric ratios
C Finding side lengths
D Finding angles
E Problem solving
F True bearings

Keywords:

- adjacent side
- angle of depression
- angle of elevation
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- sine
- tangent
- trigonometry
- true bearing
- true north

When introducing the trigonometric ratios in Section B, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 57°$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

In Section E (Problem solving), some questions which use circle geometry have been added. This has been done since this chapter now appears after the Circle geometry chapter.

The work on true bearings (Section F) extends what is done in MYP 4 to consider multi-leg journeys, in which students must find the bearing of the end point from the starting point. In this chapter, the bearings of each leg are chosen to create a right angled triangle. This acts as a lead-in to the more realistic and complex problems in the following chapter, where the bearings can take any value, and distances and bearings are found using non-right angled triangle trigonometry.

As with the chapter on Pythagoras' theorem, the questions involving three-dimensional objects have been absorbed into the section on problem solving, rather than being presented as a section of their own.

CHAPTER 18: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

A Trigonometry with obtuse angles
B The area of a triangle
C The sine rule
D The cosine rule
E Problem solving

Keywords:

- cosine rule
- sine rule
- unit circle

Non-right angled triangle trigonometry is only marked as Extended content in the MYP Framework, but we would encourage classes to complete this chapter, since non-right angled triangle trigonometry is part of all of the courses in the Diploma Programme. The concept of extending the definition of trigonometric ratios beyond acute angles is likely to be quite challenging for students, and if students can at least be introduced to this concept in MYP 5, it will help them for their future study in trigonometric functions and identities.

In Section A we define the trigonometric ratios for obtuse angles. It may be beneficial to students to see that, although it is not meaningful to talk about the trigonometric ratios for obtuse angles based on our original definition involving right angled triangles, we can extend our definition of trigonometric ratios in a way that is consistent with our original definition, but is also meaningful for a greater range of angles. This is similar to what we did in Chapter 1, when we gave meaning to zero and negative exponents.

In Section A we also establish the relationship between the trigonometric ratios of supplementary angles, which will be useful when considering the sine and cosine rules later in the chapter.

In Section B (The area of a triangle), Question **2** should help students see that the standard formula for the area of a triangle $A = \frac{1}{2} \times \text{base} \times \text{height}$, is a special case of the formula given here, in which the included angle is a right angle.

In Section C.2, students use the sine rule to find unknown angles, and will encounter the ambiguous case. It may be useful for students to consider a situation where we would use the sine rule to find an angle, in which we know two sides of a triangle, and a non-included angle. From their work in congruence, students should realise that this is not necessarily enough information to describe a triangle uniquely. This should help them understand why the ambiguous case arises.

In Section D (The cosine rule), students may benefit from seeing that Pythagoras' theorem is a special case of the cosine rule, in which the included angle is a right angle.

In Section E (Problem solving), students were introduced to true bearings in Chapter 17, so no further explanation of them is given here. The questions in this chapter allow a greater range of scenarios to be considered, where the cosine and sine rules are used to find distances and bearings.

CHAPTER 19: PROBABILITY

A Sample space and events
B Theoretical probability
C The addition law of probability
D Independent events
E Dependent events
F Experimental probability
G Expectation
H Conditional probability

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complementary events
- compound events
- conditional probability
- dependent events
- disjoint
- event
- expectation
- experimental probability
- frequency
- impossible event
- independent events
- mutually exclusive
- probability
- relative frequency
- sample space
- theoretical probability
- tree diagram

The contents of this chapter have been restructured to mirror the structure used in MYP 4.

Section A (Sample space and events) considers not only the different ways to represent the sample space of an experiment, but also the definition of an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, justifying its inclusion at MYP 5.

In Section B (Theoretical probability), the method for finding unknown numbers in Venn diagram regions has been updated to reflect the more formal method described in Chapter 6.

In the Puzzle at the end of Section B, students should be able to recognise that an easy way to solve the puzzle is to add 1 to each face of one die (so it is numbered 2 to 7), and subtract 1 from each face of the other die (so it is numbered 0 to 5). However, students who find this solution should be challenged to find a less trivial solution; one that involves the same number appearing on a die more than once.

Section C (The addition law of probability) is effectively a restructure of Section F (Mutually exclusive and independent events) from the previous edition. Rather than first considering mutually exclusive events and then generalising to the addition law, here we focus on the addition law of probability, and then observe how the law changes in the special case that A and B are mutually exclusive events. In this case $\mathrm{P}(A \cap B) = 0$, and so $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$. The remaining theory about independent events was removed, and some relevant questions were moved to Section D.

Section G (Expectation) has been added in this edition. We feel that this is an important topic to address at MYP 5, since all of the Diploma Programme courses contain some more advanced study of expectation and expected value.

We have also added Section H (Conditional probability) in this edition. The way conditional probability is approached here is quite intuitive, and does not involve any formal formulae.

CHAPTER 20: STATISTICS

A Discrete numerical data
B Continuous numerical data
C Describing the distribution of data
D Measures of centre
E Box plots
F Cumulative frequency graphs

Keywords:

- bimodal
- bimodal distribution
- box plot
- categorical data
- categorical variable
- census
- class interval
- column graph
- continuous numerical variable
- cumulative frequency
- cumulative frequency graph
- discrete numerical variable
- distribution
- dot plot
- five-number summary
- frequency histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value
- modal class
- mode
- negatively skewed distribution
- numerical data
- numerical variable
- outlier

- parallel box plot
- percentile
- population
- positively skewed distribution
- range
- sample
- statistics
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

This chapter has been moved much later in the book so that it is grouped together with probability and bivariate statistics.

The chapter opens in much the same way as the previous edition. However, we have moved the material on "Describing the distribution of data" to its own section to match what we did for MYP 4.

The "Demographics" Global Context on page 351 is a good opportunity to practise skills from both probability and statistics in the context of data for human populations.

Cumulative frequency graphs have been moved after box plots as we feel that it is better to introduce quartiles first to provide:

- a gentler introduction to percentiles
- more motivation for *using* cumulative frequency graphs.

CHAPTER 21: BIVARIATE STATISTICS

A Scatter graphs
B Correlation
C Pearson's correlation coefficient r
D Line of best fit by eye
E Linear regression

Keywords:

- bivariate data
- bivariate statistics
- correlation
- dependent variable
- extrapolation
- independent variable
- interpolation
- least squares regression line
- linear model
- linear regression
- line of best fit by eye
- mean point
- Pearson's correlation coefficient
- pole
- scatter graph

The "Scatter graphs" section has been adapted to place a greater emphasis on the construction of scatter graphs and interpretation of the data as a whole. We felt that identifying individual points did not fit the overall theme of the chapter, and students would already be familiar with identifying points on a number plane from their studies of coordinate geometry.

Material on the coefficient of determination r^2 has been removed as it is not part of the course. Exercise questions have been adjusted to use the r-value for measuring the correlation between the variables instead.

Most of the skills and concepts required for the "Capture-recapture method" Global Context on page 380 are covered in Chapters 19 and 20 ("Probability" and "Statistics" respectively). However we have placed it in this chapter as one of the later questions requires linear regression.

CHAPTER 22: RELATIONS AND FUNCTIONS

A Relations and functions
B Function notation
C Domain and range
D Sign diagrams
E Transformations of graphs

Keywords:

- critical value
- domain
- function
- function notation
- function value
- image
- interval notation
- natural domain
- range
- reflection
- relation
- scale factor
- sign diagram
- stretch
- transformation
- translation

This is intended to be a relatively simple chapter, and students will probably find it a welcome change from the previous few chapters. The purpose of this chapter is to put in place the terminology and structures that will be encountered when studying quadratic functions and exponential functions in later chapters.

In this edition, we have presented domain and range in their own section to give them more emphasis, rather than being part of the Relations section. Section A now focuses on distinguishing between a relation and a function. This structure mirrors what is done in the corresponding chapters of the Diploma Programme courses. This also allows us to place the subsection about the natural domain of a function in Section C (Domain and range) rather than Section B (Function notation).

Section D (Sign diagrams) has been moved here from Chapter 23 (Inequalities) in the previous edition. We have included an Investigation in which students discover how the form of the factors of a function affect its sign diagram. Sign diagrams will be used to describe stationary points in Chapter 26 (Differential calculus).

Section E (Transformations of graphs) has been added in this edition. Students apply translations, stretches, and reflections to a function to obtain the graph of a related function.

In the Discussion at the end of Section E, students should conclude that:

- No points are invariant when a function is translated, since all points move the same distance in the same direction.
- When a function is stretched vertically or reflected in the x-axis, points on the x-axis are invariant.
- When a function is stretched horizontally or reflected in the y-axis, points on the y-axis are invariant.

CHAPTER 23: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Axis of symmetry of a quadratic
F Vertex of a quadratic
G Finding a quadratic function
H Problem solving

Keywords:

- axis of symmetry
- completed square form
- completing the square
- maximum value
- minimum value
- parabola
- quadratic function
- turning point
- vertex
- x-intercept
- y-intercept

In Section B, we have added an introductory Investigation in which students use the geometric definition to generate a parabola. There is also an Investigation at the end of the section which explores one of the properties of parabolas. Students should find that all of the rays reflected off the parabola pass through a single point, which is the focus of the parabola.

In Section C (Using transformations to graph quadratics), rather than having a large Investigation to explore the graphs of quadratic functions, we can use the work done on transformations in the previous chapter to first establish some basic ideas about the shape and position of $y = (x-h)^2 + k$ and $y = -x^2$. We then investigate the effect of the value of a on the shape of the graph, and the direction in which it opens.

In Questions **7** to **9** of Exercise 23C.1, students must verify the completed square form of each quadratic function, then use this form to sketch the graph. In Section C.2, students must convert the quadratic functions into completed square form themselves. This subsection should only be done by students who used completing the square to solve quadratic equations in Chapter 12.

Section G (Finding a quadratic function) has been added in this edition. Finding a function based on information about its graph is an important modelling skill which students may find useful in later years, especially those looking to study one of the Mathematics: Applications and Interpretation courses in the Diploma Programme.

The “Quadratic optimisation” section has been renamed “Problem solving” (Section H), since students are asked to solve problems other than identifying the optimum point. The Global Context at the end of this section provides some interesting examples of the use of parabolas and circles in the construction of arches.

CHAPTER 24: NUMBER SEQUENCES

- **A** Number sequences
- **B** Arithmetic sequences
- **C** Geometric sequences
- **D** Sequences in finance

Keywords:

- arithmetic sequence
- common difference
- common ratio
- compound interest
- depreciation
- explicit formula
- general term
- geometric sequence
- initial condition
- nth term
- number sequence
- recurrence relation
- sequence
- series
- term

The work in this chapter is only part of the Extended content in the MYP Framework, and may be skipped if classes are short on time. However, we would recommend that this chapter be completed if time permits, since sequences are studied in all of the Diploma Programme courses.

Section A provides an introduction to the notation and terminology associated with number sequences. We describe sequences by listing the terms, writing the sequence in words, or by writing an explicit formula or recurrence relation.

Although recurrence relations are not mentioned explicitly in the MYP Framework, we inherently use them when describing a sequence in words, so there is value in learning to describe them algebraically. In the Discussion in Section A, students should conclude that we can also write a description for a sequence in words which is linked to the explicit formula. For example, a sequence with explicit formula $u_n = 3n$ can be described in words as “the nth term of the sequence is equal to 3 times n”.

Sections B (Arithmetic sequences) and C (Geometric sequences) will provide a useful introduction to the sequences work done in any of the Diploma Programme courses.

We have converted the “Sequences in finance” Investigation from the previous edition into a section of its own (Section D). Studying them here, in the context of sequences, will better mirror what is done in the Diploma Programme courses.

The main difficulty here is understanding that we consider the initial condition to represent the “zeroth” term u_0. We do this so that the situation after n time periods is represented by u_n, rather than u_{n+1}.

The work done here will also help in understanding exponential functions in the following chapter. Even though exponential functions deal with continuous time intervals, the idea of exponential growth is more intuitively described by thinking in terms of discrete time intervals, such as what is done here for compound interest and depreciation. For example, an exponential function $f(x) = 5 \times 1.2^x$ tells us that as x increases by 1, the value of the function increases by 20%.

This chapter contains two Global Context projects. Although both projects involve number sequences, they are quite different to each other. The golden ratio Global Context at the end of Section A guides students through some useful properties of the golden ratio, as well as discussing its occurrence in the world around us. This project discusses the link between the Fibonacci sequence and the golden ratio, so students may benefit from completing the preceding Investigation about the Fibonacci sequence. The spider webs Global Context at the end of Section C examines the geometric properties of spider webs constructed according to particular rules. In order to determine an “optimal” design for a spider web, students must analyse a large table of data, which is likely to produce some surprising results.

The work on series in the previous edition has been removed, as it is only mentioned under Enrichment in the Framework.

CHAPTER 25: EXPONENTIALS

A Exponential functions
B Graphs of exponential functions
C Exponential equations
D Exponential growth
E Exponential decay

Keywords:

- exponential decay
- exponential equation
- exponential function
- exponential growth
- horizontal asymptote
- y-intercept

This will be the first time students have encountered exponential functions. Students should understand that, while linear functions are characterised by a quantity changing by a constant *amount* each time period, exponential functions are characterised by a quantity changing by a constant *percentage* each time period.

It may help students to look back at the work on sequences in finance, and see that the formula for the value of a compound interest account, or the value of a depreciating item, was an exponential function, which increased or decreased by a certain percentage each time period. For example, a compound interest account which increases by 5% each year grows in value according to the exponential function $V(t) = V_0 \times 1.05^t$.

This is why number sequences are presented before exponential functions, even though these geometric sequences are not part of the Standard course. If classes wish to skip the number sequences chapter, a little extra time must be spent explaining the link between the a in a^x and the rate of growth.

Now that a section about transformations of graphs has been added, we no longer need a large investigation about using transformations to graph exponential functions. We simply apply the general principles we studied to exponential functions in Section B.

In this edition, we have moved exponential equations ahead of exponential growth and decay. The exponential equations section now includes practice at solving exponential equations using technology. This can either be done graphically, or by using the Solver function of the calculator. This can then be used in exponential growth and decay to answer questions such as "How long will it take for the population to reach 500?".

CHAPTER 26: DIFFERENTIAL CALCULUS

A Limits
B Finding the gradient of a tangent
C The derivative function
D Differentiation from first principles
E Rules for differentiation
F Finding the equation of a tangent
G Stationary points

Keywords:

- calculus
- converge
- derivative
- derivative function
- differential calculus
- differentiation
- differentiation from first principles
- gradient function
- instantaneous rate of change
- limit
- local maximum
- local minimum
- stationary inflection
- stationary point
- tangent
- turning point

In this edition, the calculus material has been split into two chapters: Differential calculus, and Integration. This material is not part of the MYP Framework, so we have marked it dark blue, and placed it online. However, calculus plays a large role in all of the Diploma Programme courses, so if students are able to complete these chapters in MYP 5, they would be at a huge advantage going into their Diploma Programme courses.

This chapter essentially covers what was in Sections A to E of Chapter 27 (Introduction to calculus) in the previous edition. This work has been restructured to match the structure used in the calculus chapters in the Diploma Programme courses.

The work that was previously in Section A (Tangents) about estimating the gradient of a tangent from a graph has been converted into an Activity before Section A, since we felt that it was not really the focus of the chapter. This Activity motivates the idea that, in order to find the *exact* gradient of a tangent for a function, we need the mathematical principle called a limit, which is explored in Section A.

In Section B (Finding the gradient of a tangent), we use limits to find the gradient of a tangent to a function at a particular point. This work appeared in Section 27B.2 of the previous edition.

The material in Section C (The derivative function) is mainly new, and is aimed at familiarising students with the notation surrounding the derivative function, such as $f'(x)$ or $f'(2)$. The material in the section of the same name in the previous edition was concerned with finding the derivative function from first principles. This work is done in Section D.

We have added Section F (Finding the equation of a tangent) in this edition. We feel that once students can find the gradient of a tangent, it is logical to take the next small step to find its equation.

CHAPTER 27: INTEGRATION

A The area under a curve
B Integration
C Rules for integration
D The definite integral
E The Riemann integral

Keywords:

- antiderivative
- antidifferentiation
- constant of integration
- definite integral
- integral
- integral calculus
- integration
- Riemann integral

This chapter essentially covers what was in Sections F to H of Chapter 27 (Introduction to calculus) in the previous edition.

The Opening Problem should lead students to the idea that we can estimate the area under a curve using rectangles to approximate the region. In Section A, we extend this idea and see how limits can be used to find the area exactly.

In Section B, we define integration as the reverse process of differentiation, and use this to find the integral of particular functions. This work was covered in Section 27G.1 in the previous edition. This process leads to the establishment of some general rules for integration (Section C), which was covered in Section 27G.2 in the previous edition. In this edition, we have distinguished between the rules which follow directly from the rules of differentiation, and the more general rules about integrating the sum or difference of two functions, or a function multiplied by a scalar.

We have divided the material that was in Section 27H (The definite integral) in the previous edition into two sections. Section D defines the definite integral and introduces the notation associated with it. Section E looks at the interpretation of the definite integral as the area under the curve.