Mathematics for Australia Year 9 2nd edition

Chapter summaries

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June 9, 2022

ASSUMED KNOWLEDGE: NUMBER

- **A** Prime and composite numbers
- **B** Highest common factor
- **C** Multiples
- **D** Operations with fractions
- **E** Rounding numbers
- **F** Roots
- **G** Rational numbers

Keywords:

- common fraction
- composite number
- cube root
- decimal places
- denominator
- equal fractions
- factor
- fraction
- highest common factor
- improper fraction
- lowest common denominator
- lowest common multiple
- mixed number
- multiple
- numerator
- place value
- prime number
- proper fraction
- radical
- rational number
- reciprocal
- recurring decimal
- round
- significant figures
- square root
- terminating decimal

This online chapter has been included for students who feel they need extra help with the basic properties of numbers. Students who need this help should be encouraged to study the chapter out of classroom time, since most students will not need this chapter, and classroom time spent on this chapter is likely to come at the expense of time spent on the more advanced chapters at the end of the year.

CHAPTER 1: NUMBER

- **A** Exponent notation
- **B** The Fundamental Theorem of Arithmetic
- **C** Order of operations
- **D** Absolute value

Keywords:

- absolute value
- base
- BEDMAS
- counting number
- exponent
- exponent notation
- factor tree
- index
- integer
- natural number
- power
- prime factored form
- prime factorisation
- prime number
- real number

- repeated division
- whole number

This chapter has been added to the book for this new edition. Section A has largely been taken from the start of Chapter 2 (Indices) in the previous edition. We now use the term "exponent" rather than "index", as this is the terminology used in the most recent update to the Australian Curriculum.

The inclusion of this Number chapter allows us to study absolute value, which will lead to the study of absolute and percentage error in Chapter 4.

Given that there is no dedicated fractions chapter in Year 9, we also have the opportunity to revise the order of operations here, without the need to leave the fractions material until a later chapter. This material should be familiar to most students, and may be skipped if this is the case.

CHAPTER 2: ALGEBRA: EXPRESSIONS

Keywords:

- algebra
- algebraic fraction
- algebraic product
- algebraic quotient
- coefficient
- constant
- equation
- evaluate
- exponent notation
- expression
- highest common factor
- like terms
- product notation
- substitute
- term
- variable

This chapter has been adapted from what was Chapter 1 in the previous edition. In this edition, Section D (The language of algebra) has been included to reinforce the terminology introduced in Year 8.

In the Discussion at the end of Section B, students should find that the difficulty in talking about the difference between 5 and x is that, without knowing the value of x, we do not know which number is greater. So, the difference may be $x-5$, or $5-x$. By writing the difference between 5 and x as $|5-x|$, we can avoid this difficulty and guarantee that our difference is positive.

Algebraic quotients are dealt with in this chapter, but in this edition it is only in the context of simplifying quotients by cancelling common factors. Operations with algebraic fractions have been moved to a chapter of their own later in the book (Chapter 16).

Algebraic common factors have been added to the end of this chapter to mirror what was done in Year 8. This work will be useful when operating with algebraic fractions in Chapter 16, and when factorising algebraic expressions.

CHAPTER 3: EXPONENTS

Keywords:

- derived unit
- exponent laws
- International System of Units
- negative exponent law
- scientific notation
- standard form
- zero exponent law

In this edition, we have changed the name of this chapter from “Indices” to “Exponents” to reflect the changed terminology in the most recent Australian Curriculum update.

“Evaluating indices” has been moved to Chapter 1 (Number), and is now called “Exponent notation”. This appears to be a more sensible place to introduce exponent notation, given that expressions involving exponents are presented in both Chapters 1 and 2.

In the Discussion at the end of Section B, students should realise that cancelling the common factor of x is problematic if $x = 0$, since we cannot divide by 0. Students should be able to use the exponent laws to show that $\frac{x}{x} = x^0$. Given that any non-zero number divided by itself is 1, this is reasonable justification for concluding that $a^0 = 1$ for $a \neq 0$. However, when $x = 0$, we have $\frac{0}{0} = 0^0$. Since $\frac{0}{0}$ is undefined, this leads us to the conclusion that 0^0 is undefined.

We have adjusted our approach to explaining scientific notation, to give students more guidance in how to choose their values of a and k in $a \times 10^k$.

We have added a new section “International System (SI) units”, in which we use prefixes such as “micro”, “nano”, “mega”, and “giga”, in conjunction with scientific notation, to write very large and very small quantities.

CHAPTER 4: PERCENTAGE

A Converting percentages into decimals and fractions
B Converting decimals and fractions into percentages
C Expressing one quantity as a percentage of another
D Finding a percentage of a quantity
E The unitary method for percentages
F Percentage increase or decrease
G Finding a percentage change
H Absolute and percentage error
I Finding the original amount
J Simple interest
K Compound interest

Keywords:

- absolute error
- compound interest
- interest rate
- multiplier
- percentage
- percentage error
- principal
- simple interest
- unitary method

The basics of percentage are reviewed in Sections A to D, so classes should move through these sections quickly if students are familiar with the content.

This will be the first time students encounter the unitary method (Section E) and finding the original amount (Section I), so these sections should be worked through more carefully.

In the Discussion at the end of Section E, students should find that the process can be made quicker by multiplying by a single fraction. For example, suppose 17% of a quantity is known, and we need to find 60% of the quantity. Rather than dividing by 17 to find 1%, then multiplying by 60 to find 60%, the desired quantity can be found in one step by multiplying by $\frac{60}{17}$.

We have added a section on absolute and percentage error (Section H), as specified in the new curriculum. In the Discussion at the end of this section, students should find that, if an estimate is too low, the percentage error must be between 0% and 100%, whereas, if an estimate is too high, there is no upper limit on the percentage error. When thinking about situations in which absolute error or percentage error may be more appropriate, students could think about the estimates of π in Question **5** of Exercise 4H. Since each of these estimates are estimates of the same number, we could just as easily have compared the estimates using only the absolute error. However, if we wanted to compare an estimate of π with, for example, an estimate of $\sqrt{2}$, it would not be fair to compare the absolute errors. This is because $\sqrt{2}$ is smaller than π, so any estimate of $\sqrt{2}$ is likely to have a smaller absolute error. A more appropriate way to compare the accuracy of the two estimates would be to compare their percentage errors.

CHAPTER 5: ALGEBRA: EXPANSION

- **A** The distributive law
- **B** The product $(a+b)(c+d)$
- **C** The difference between two squares
- **D** The perfect squares expansion
- **E** Further expansion

Keywords:

- difference between two squares
- distributive law
- expansion
- FOIL rule
- perfect squares

This chapter has been adapted from Chapter 5 (Algebraic expansion and simplification) in the previous edition. Students should have encountered the distributive law (Section A) in Year 8, but the rest of the chapter will be new to them.

Students should be encouraged to see the difference between two squares expansion, and the perfect squares expansion, as special cases of the $(a+b)(c+d)$ expansion. Questions **3** and **4** of Exercise 5B should help with this.

We have removed material on the binomial expansion in this edition, as it is not part of the syllabus. The binomial expansion will be studied in Year 10.

CHAPTER 6: SETS

- **A** Sets
- **B** Complement of a set
- **C** Intersection and union
- **D** Special number sets
- **E** Interval notation

Keywords:

- complement
- complementary set
- disjoint
- element
- empty set
- equal sets
- infinite set
- integer
- intersection
- interval notation
- irrational number
- member
- natural number
- negative integer
- positive integer
- rational number
- real number
- set
- set notation
- subset
- union
- universal set

In this edition, we have split the Sets and Venn diagrams chapter into two separate chapters. In the previous edition, students were asked to solve linear equations when finding unknown numbers of elements in Venn diagrams, before they reached the Linear equations chapter.

With the introduction of linear inequalities at Year 9, we felt the best approach was to split Sets and Venn diagrams into two chapters, with "Linear equations and inequalities" in between them. This way, interval notation can be intoduced in Sets before it is used in linear inequalities, and linear equations can be used to solve problems involving Venn diagrams.

We have introduced the concept of sets more slowly in this edition. For example, rather than including intersection and union in Section A, intersection and union are presented in their own section in Section C. This allows students to become comfortable with the concepts and terminology of sets in Section A, before we perform operations with them.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

In the Discussion at the end of Section C, it would be helpful for students to experiment with cases where one of the unknown numbers is a multiple of the other, and cases where neither number is a multiple of the other. Students should determine that there will always be sufficient information to determine the values of the two numbers. They should find that the largest value in $X \cup Y$ gives us one of the numbers (the larger number). If all of the elements of $X \cup Y$ are factors of this number, this tells us that the smaller number is a factor of the larger number, in which case the largest value of $X \cap Y$ gives us the smaller number. If there are values in $X \cup Y$ which are *not* factors of the larger number, then the largest of these values gives us the smaller number.

In Section D, we have changed the definition of natural numbers to include 0. Although either interpretation is accepted in the Australian Curriculum glossary, this appears to be the most commonly used definition in other curriculums around the world.

In the Discussion at the end of Section D, students should find that all of the examples given *may* be rational. For example:

- $\pi + (-\pi) = 0$, which is rational.
- $(\sqrt{3} + 5) - \sqrt{3} = 5$, which is rational.
- $\sqrt{2} \times \sqrt{2} = 2$, which is rational.

In the Discussion at the end of Section E, it would be useful for students to think about the difficulties inherent in representing the rational numbers on a number line. It is not so much that there are infinitely many rational numbers, since we can easily represent an interval of infinite real numbers using a solid line. The issue is that between any two rational numbers there are irrational numbers, so any representation of rational numbers would have to consist of an infinite number of disconnected dots, which would be impractical.

CHAPTER 7: LINEAR EQUATIONS AND INEQUALITIES

A Linear equations
B Equations with fractions
C Problem solving
D Linear inequalities
E Solving linear inequalities

Keywords:

- algebraic equation
- equal sign
- equate numerators
- equation
- inequality
- inequality sign
- inverse operation
- left hand side
- linear equation
- linear inequality
- lowest common denominator
- right hand side
- solution

In this edition, we have added linear inequalities to this chapter, as it is now a part of the syllabus at Year 9.

Section A is largely revision of the work done on solving linear equations in Year 8, and may be skipped through more quickly if students are comfortable with the material. Section B is more likely to be challenging for students.

The mixture problems which were in Section D of the previous edition have been moved to a subsection of Problem solving (Section C).

One of the biggest conceptual difficulties when moving from equations to inequalities is the idea that the solution is no longer a single value of x, but an interval of infinitely many values of x. It may help students to consider a particular inequality and, by substitution, show that there are many values of x which satisfy the inequality.

When solving linear inequalities, students can use the work done in Chapter 6 to write the solutions in interval notation, and to graph the solution on a number line.

CHAPTER 8: VENN DIAGRAMS

A Venn diagrams
B Venn diagram regions
C Numbers in regions
D Problem solving with Venn diagrams

Keywords:

- complement
- disjoint
- intersection
- number of elements
- subset
- union
- Venn diagram

This chapter presents the remainder of what was the "Sets and Venn diagrams" chapter in the previous edition. By placing linear equations between the chapters, students can use their linear equation solving skills to find unknown numbers in regions on Venn diagrams.

Section A may be worked through quickly if students are comfortable with this material from Year 8.

Section C (Numbers in regions) has been moved from being a subsection of the Venn diagrams section, to being a section in its own right, to mirror what was done in Year 8. Some more complex cases are considered here in Year 9, and having linear equations behind them allows students to apply a more methodical approach to find the unknown numbers.

CHAPTER 9: SURDS AND OTHER RADICALS

A Square roots
B Properties of radicals
C Simplest surd form
D Cube and higher roots
E Power equations

Keywords:

- cube root
- nth root
- power equation
- radical
- simplest surd form
- square root
- surd

This chapter builds on what was in Sections A, B, and H of the "Radicals and Pythagoras" chapter in the previous edition.

Making this material into its own chapter allows us to extend the work beyond the square root work needed for Pythagoras, to further consider cube and higher roots, as well as other power equations such as $x^3 = k$. We feel that it is important to consider power equations, as they are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right.

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$. In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations.

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form.

CHAPTER 10: PYTHAGORAS' THEOREM

A Pythagoras' theorem
B Pythagorean triples
C Problem solving
D The converse of Pythagoras' theorem

Keywords:

- converse of Pythagoras' theorem
- hypotenuse
- Pythagoras' theorem
- Pythagorean triple
- right angled triangle

Moving the work on radicals to a chapter of its own for this edition allows us to focus exclusively on Pythagoras' theorem in this chapter.

The Investigation which was previously in the Pythagorean triples section has been converted into an Activity, and moved to the end of Section A. The Activity is now more general, and students use Pythagoras' theorem to plot irrational numbers on the number line. This Activity also aligns better with an outcome outlined in the new Australian Curriculum.

In Section B, we have removed the work on using *multiples* to find unknown values on triangles, and replaced it with a more formal consideration of how, given a Pythagorean triple, we can multiply each element by a positive integer to obtain another Pythagorean triple.

In this edition, we have changed the order of the sections, so that the converse of Pythagoras' theorem is done at the end. This allows us to consider the non-contextual and contextual applications of Pythagoras' theorem first, before considering the converse.

CHAPTER 11: FORMULAE

- **A** Formula construction
- **B** Substituting into formulae
- **C** Rearranging formulae
- **D** Rearrangement and substitution

Keywords:

- formula
- inverse operation
- rearrange
- subject
- substitute

In this edition, we have moved the Formulae chapter before the measurement chapters, as we felt that students will benefit from doing this work before encountering the measurement formulae. As a result, we have removed any questions referring to measurement formulae, as these will be addressed in the measurement chapters. In their place, we have included more questions based on physics formulae, as specified in the new Australian Curriculum.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 9 should help with this.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

CHAPTER 12: MEASUREMENT: LENGTH AND AREA

- **A** Units of length
- **B** Perimeter
- **C** Units of area
- **D** Area of polygons
- **E** Area of circles and sectors

Keywords:

- arc length
- area
- centimetre
- circumference
- hectare
- kilometre
- metre
- millimetre
- perimeter
- sector

As in previous years, we have removed perimeter formulae for specific polygons on the basis that it is more important to understand what the perimeter is, at which point the formulae are not helpful.

Now that this chapter occurs after the Formulae chapter, we have included more questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

We have divided the area formulae section into two sections. Section D involves the area of polygons, all of which the students should have seen in previous years. Given that the formulae should be familiar to students, we are able to provide some more challenging questions within this section. Section E is focused on the area of circles and sectors, which students may be less familiar with.

The "Areas of composite figures" section from the previous edition has been removed, and instead such questions have been inserted into the appropriate sections. The idea of finding the area of a composite shape by adding and subtracting known areas is a reasonably intuitive idea, and we feel that it can be explained by a worked example at the appropriate point in the exercise, without the need to dedicate a whole section to it.

CHAPTER 13: MEASUREMENT: SURFACE AREA

A Solids with planar faces
B Cylinders
C Cones
D Spheres

Keywords:

- net
- surface area

This short chapter is adapted from Section B of the Further measurement chapter in the previous edition. Sections A (Solids) and B.1 (Nets of solids) of the Further measurement chapter have been removed in this edition, since students should be familiar with this work from previous years.

Students should have seen the surface area work in Section A and B in Year 8. The main difference here is the use of Pythagoras' theorem to find unknown lengths in some solids.

The surface area of a cone and a sphere are not part of the Year 9 syllabus, but we felt it useful for students to see them at this stage. Introducing surface area in Year 8 in this series allows us to push a little further in Year 9. This in turn avoids the need to introduce many new measurement formulae in Year 10, when there is so much other content to cover.

In Section C, students are guided through an Investigation in which the net of a cone is used to derive its surface area.

In Section D, since it is not possible to draw a net for a sphere, students explore the relationship between the area of the curved surface and the flat surface of a hemisphere. Students should find that the area of the curved surface of the hemisphere is approximately twice the area of the flat surface, so the area of the whole sphere is given by $A = 4\pi r^2$. It should be emphasised to students that this is not a strict mathematical proof of the formula for the surface area of sphere, but rather a demonstration to illustrate that the formula is reasonable.

CHAPTER 14: MEASUREMENT: VOLUME AND CAPACITY

A Units of volume
B Volume of a solid of uniform cross-section
C Volume of a tapered solid
D Volume of a sphere
E Capacity
F Connecting volume and capacity

Keywords:

- apex
- capacity
- kilolitre
- litre
- megalitre
- millilitre
- solid of uniform cross-section
- tapered solid
- volume

This chapter is adapted from Sections C and D of the Further measurement chapter in the previous edition.

The material in Sections A and B should be familiar to students from Year 8. In Year 9, a greater emphasis is placed on finding a dimension, given the volume of a solid.

In Activity 2 at the end of Section B, students explore the ratio surface area : volume for prisms of various shapes. In general, students should find that, for a given volume, we can make a large surface area by making the prism extremely long and flat, whereas we can make the surface are smaller by making the prism more "regular", with sides of similar length. For example, a cube of length 1 m and a rectangular prism measuring $10 \text{ m} \times 10 \text{ m} \times 1 \text{ cm}$ each have volume 1 m^3. However, the cube has surface area 6 m^2, whereas the rectangular prism has surface area 200.4 m^2.

The volumes of tapered solids and cubes are not part of the Year 9 syllabus, but, as with surface area, we have decided to include them here to avoid introducing lots of new measurement formulae in Year 10. These sections may be skipped if classes are running short on time, however this will mean extra time will need to be spent on these formulae in Year 10,

In Section C, students are guided through an Investigation in which they compare the volume of a tapered solid with the volume of the corresponding solid with uniform cross-section. Students should find that the volume of the tapered solid has one third the volume of the corresponding solid with uniform cross-section.

Capacity was skipped in Year 8, in order to wait until the students have more volume formulae to work with in Year 9. So, students may need some extra time to familiarise themselves with the connection between volume and capacity.

CHAPTER 15: ALGEBRA: FACTORISATION

A Common factors
B Difference between two squares factorisation
C Perfect squares factorisation
D Quadratic trinomials
E Miscellaneous factorisation

Keywords:

- factorisation
- fully factorised
- quadratic trinomial
- sum and product factorisation

The material on finding highest common factors in algebraic expressions has been moved to Chapter 2 (Algebra: expressions). This is a more logical place for the material, and it means that we do not need to be side-tracked in this factorisation chapter.

To further emphasise that factorisation is the reverse process of expansion, students should be reminded that they can check their factorisations by expanding their answer.

In Section E, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable. It may be helpful for them to realise that the difference between two squares factorisation and perfect squares factorisation are special cases of the sum and product factorisation (in difference between two squares, the required numbers p and q are negatives of each other, and in perfect squares factorisation, the numbers are the same).

CHAPTER 16: ALGEBRAIC FRACTIONS

A Evaluating algebraic fractions
B Simplifying algebraic fractions
C Multiplying algebraic fractions
D Dividing algebraic fractions
E Adding and subtracting algebraic fractions

Keywords:

- algebraic fraction
- evaluate
- factorise
- highest common factor
- rational expression

This chapter has been expanded from what was Section F (Algebraic fractions) of Chapter 1 (Algebra) in the previous edition. Some very basic simplification still occurs in Section 2G (Algebraic quotients) of this edition, but the more involved simplifications and operations have been expanded into this chapter.

Placing this chapter after we have studied factorisation allows us to factorise the numerator and denominator of an algebraic fraction. This helps in cancelling common factors when simplifying, multiplying, or dividing algebraic fractions.

In Section E, we have avoided using the term “simplify” as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the “simplest” in this case. Instead, we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term “simplify”, and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is “better” or “simpler” may depend on what we are trying to do with the expression.

CHAPTER 17: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient
D Parallel and perpendicular lines
E Using coordinate geometry

F 3-dimensional coordinate geometry

Keywords:

- axis
- Cartesian plane
- coordinates
- distance formula
- gradient
- midpoint
- negative reciprocal
- ordered pair
- origin
- parallel lines
- perpendicular lines
- quadrant
- x-axis
- X-axis
- x-coordinate
- y-axis
- Y-axis
- y-coordinate
- Z-axis

In this edition, the material involving the equations of lines has been moved to its own chapter "Straight lines" (Chapter 18). In the content that remains, students explore the Cartesian plane, including distance between points, midpoints, and gradients. We feel this is a logical way to divide this material, since some new material has been added, and a single chapter would be very large. The work in this chapter provides students the tools to describe straight lines in the Cartesian plane in the following chapter.

While students encountered the gradient of a line in Year 8, it was only in terms of horizontal and vertical steps. In Year 9, students are given the gradient formula. In the Discussion in Section C, it would be useful to remind students that the gradient is a measure of the *steepness* of a line. In this sense, it is desirable that a steeper line has a higher gradient.

The gradient of parallel and perpendicular lines, which was only presented in an Investigation in the previous edition, is now presented as a section of its own, since it has been added to the syllabus at Year 9. In the Discussion in Section D, students should find that the rules for perpendicular lines do not apply to horizontal and vertical lines, and that these are special cases which will need to be treated separately. In other words, the idea that vertical and horizontal lines are perpendicular should be intuitive, and gradient is not a very useful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry), which is a new section for this edition.

Section F (3-dimensional coordinate geometry) has also been added in this edition, as it has been included in the Australian Curriculum. Students should be reminded that, although it is harder to visualise coordinates in three dimensions, most of the work that is done (such as finding distances and midpoints) extends fairly logically from what they have seen in two dimensions.

CHAPTER 18: STRAIGHT LINES

A Vertical and horizontal lines
B Points on a line
C Axes intercepts
D Graphing from a table of values
E Gradient-intercept form
F General form
G Finding the equation of a line

Keywords:

- axes intercepts
- equation of a line
- general form
- gradient-intercept form
- horizontal line
- point-gradient form
- table of values
- vertical line
- x-intercept
- y-intercept

This new chapter comprises the material about straight lines that was in Sections D to H of the Coordinate geometry chapter in the previous edition. Presenting this work as a chapter in its own right allows us to provide a more complete treatment of this material, without creating a chapter that is too large and encompassing too many different ideas.

Rather than leaving vertical and horizontal lines as an afterthought, they are presented in Section A in this edition. In Sections C and D, students have the opportunity to relate the y-intercept and gradient of a line with its equation. This lays the groundwork for graphing a line in gradient-intercept form in Section E. The work done in Section C is also used in Section F, where students use axes intercepts to graph lines in general form.

In Section G, we have more clearly delineated the combinations of information which can be provided in order to determine the equation of a line (gradient and y-intercept, gradient and a point, two points). Breaking these into separate subsections will allow students to focus on the nuances of individual questions within each subsection (such as whether the information is given in word form or graphical form, and whether the equation must be given in gradient-intercept or general form).

CHAPTER 19: SIMULTANEOUS EQUATIONS

A Solution by trial and error
B Graphical solution
C Solution by equating values of y
D Solution by substitution
E Solution by elimination
F Problem solving with simultaneous equations

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution
- trial and error

This chapter starts by asking students to solve simultaneous equations by trial and error. It is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In the Discussion at the end of Section A, the equations in **1 a** have solution $x = -\frac{1}{6}$, $y = \frac{3}{14}$, and the equations in **1 b** have no solution. The equations in **1 c** have infinitely many solutions, so it is likely that students would have found different solutions. Trying to solve simultaneous equations by trial and error makes it hard to find non-integer solutions, and it is hard to identify when a system has no solutions or infinitely many solutions. This should lead students to conclude that a more systematic approach is required.

In Section B, a graphical approach is used. This should allow students to build on what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should better illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections C to E.

CHAPTER 20: QUADRATIC EQUATIONS

A Quadratic equations
B Equations of the form $x^2 = k$
C The null factor law
D Solving by factorisation
E Problem solving
F Completing the square

Keywords:

- completing the square
- null factor law
- quadratic equation

In this edition, we have added Section B (Equations of the form $x^2 = k$). Although simple versions of these equations were solved in Section 9E (Power equations), here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In Section D.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations such as $(2x+1)^2 - (x+2)^2 = 0$, it should be clear that using difference between two squares factorisation is more efficient.

In this edition, similarity has been moved later in the book, and now appears after quadratic equations. For this reason, the questions involving similar triangles have been removed from Section E, and now appear in the Congruence and similarity chapter.

We have included Section F (Completing the square), even though it is not in the syllabus, to give more able students a chance to attempt these quadratic equations for the more simple case where $a = 1$. Completing the square will be presented again in Year 10, including cases where $a \neq 1$.

CHAPTER 21: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Using axes intercepts to graph quadratics
F Projectile motion

Keywords:

- parabola
- projectile motion
- quadratic function
- vertex
- x-intercept
- y-intercept

This chapter builds on what was Sections A to C of the Non-linear graphs chapter in the previous edition. With the graphs of circles no longer in the syllabus, this chapter is now renamed "Quadratic functions".

In this edition, we include the use of transformations to graph quadratic functions. Through an Investigation, students see how graphs of the form $y = (x-h)^2 + k$ are translations of $y = x^2$, and that graphs of the form $y = ax^2$ can be formed by stretching and reflecting the graph of $y = x^2$. We then move on to consider the axes intercepts of quadratic functions, and use the axes intercepts to graph quadratic functions, as was done in the previous edition.

Section F (Projectile motion) is largely done using technology. This is a good opportunity for students to use technology to find characteristics of the graphs such as the axes intercepts and turning points.

CHAPTER 22: CONGRUENCE AND SIMILARITY

A Congruence
B Congruent triangles
C Enlargements and reductions
D Similarity
E Similar triangles
F Problem solving
G Areas of similar figures
H Volumes of similar solids

Keywords:

- congruent figures
- congruent triangles
- enlargement
- equiangular
- reduction
- same ratio
- scale factor
- similar figures
- similar triangles

In this edition, we have presented congruence first, and then similarity. We feel this is a more logical order, firstly because students will have already seen congruence in Year 8, and secondly because similarity ends with the conceptually challenging ideas of areas and volumes of similar objects, and this seems an appropriate way to end the chapter.

We have added a short section at the start of the chapter about congruence in general, before considering the specific conditions required to prove congruence in triangles in Section B.

The Discussion at the end of Section B should cause students to realise that, just because there is not enough information to conclude two triangles are congruent, this does not *necessarily* mean that the triangles are not congruent. For example, in the first pair of triangles, we cannot conclude the triangles are congruent, as one triangle could be an enlargement of another. However, we cannot conclude the triangles are *not* congruent, since the side lengths of the triangles may indeed be equal.

In this Discussion, it may be helpful to ask students what can be inferred by a *lack* of tick marks indicating equal side lengths. For example, in the second pair of triangles, we cannot conclude the triangles are congruent, since the equal sides are not in corresponding positions. However, if the side of the second triangle between α and β is also equal to the sides with the tick marks, then the triangles *would* be congruent. Does the lack of a tick mark mean that the side lengths are definitely different?

It might also be useful to ask to what extent we can assume the diagram is drawn reasonably to scale. For example, in the third pair of triangles, we cannot conclude that the triangles are congruent, since the equal angle is not between the equal sides. However, it can be shown (once we have knowledge of trigonometry), that if we can assume the angle at the top is acute, and the angle in the middle is obtuse (as it appears in the diagram), then there is sufficient information to conclude that the triangles are congruent. Are these reasonable assumptions to make?

Students may also enjoy critiquing Lewis Carroll's "proof" that all triangles are isosceles in the Puzzle at the end of Section B. The flaw in the proof comes in *Step 1*. The point X at which the lines meet will never occur inside the triangle. If the triangle is indeed isosceles, the two lines will be coincident, and if the triangle is not isosceles, X will lie outside the triangle.

In Section E.2, some questions have been added which involve solving a quadratic equation to find the unknown. These questions were in the Quadratic equations chapter in the previous edition, but had to be moved here because the Quadratic equations chapter is now before similarity.

CHAPTER 23: TRIGONOMETRY

- **A** Scale diagrams in geometry
- **B** Labelling right angled triangles
- **C** The trigonometric ratios
- **D** Finding side lengths
- **E** Finding angles
- **F** Problem solving

Keywords:

- adjacent side
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- scale diagram
- sine
- tangent
- trigonometry

In this edition, we have added Section A (Scale diagrams in geometry). This section should be seen as making use of the similarity of a situation and its scale diagram. While these types of problems can be solved using scale diagrams, it is hard to obtain accurate answers in this manner. Trigonometry makes the process more accurate by providing the actual ratios required.

When introducing the trigonometric ratios in Section C, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 24°$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

This is likely to be the first time students have used their calculators for finding trigonometric ratios, so if their calculators are not producing the expected answers, it may be because their calculators are not set to "degree" mode. If needed, the graphics calculator instruction icon can help with this.

In the Discussion at the end of Section C, students should find that $\sin\theta$ and $\cos\theta$ can take values between 0 and 1 (a useful first step here is to recognise that the hypotenuse is the longest side of the triangle), and that $\tan\theta$ can take any positive value.

In the Puzzle at the end of Section E, students should find that 16 triangles can be drawn before they start to overlap.

CHAPTER 24: PROPORTION

- **A** Direct proportion
- **B** Powers in direct proportion
- **C** Inverse proportion
- **D** Powers in inverse proportion

Keywords:

- directly proportional
- hyperbola
- inversely proportional
- proportionality constant

In this edition, this chapter has been broken up into more sections, so that powers in direction proportion, and powers in inverse proportion, are given sections of their own.

In Example 2 of Section A, two methods are presented to find the unknown value. Students should be encouraged to see that Method 2 is the preferred approach, as it avoids the need to determine the proportionality constant k and the law connecting y and n, and instead uses the properties of the proportionality relationship. The difference in methods becomes even more pronounced as we deal with powers and with inverse proportion, and Method 2 is used in the subsequent worked examples in the chapter.

In the Discussion at the end of Section A, students should realise that there are many variables in their everyday life that are in direct proportion. For example, *distance* and *speed* are directly proportional, so if they travel twice as fast, they will travel twice the distance in a given time period.

In the Discussion at the end of Section B, students should find that the *volume* and the *mass* of the models are directly proportional to the *cube* of the *height*, while the *surface area* is directly proportional to the *square* of the height. However, we would not expect the measurements to match exactly with these relationships, since there will always be imperfections and inconsistencies with how they are made, and our measurements are likely to contain some error.

If the models are made with different materials, we would still expect the *volume* and *surface area* to have the same proportional relationships with *height* (as these do not depend on the material used). However, it is likely that there is no longer a proportional relationship between *mass* and *height*.

CHAPTER 25: PROBABILITY

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complement
- compound events
- dependent events
- event
- experimental probability
- frequency
- impossible event
- independent events
- probability
- relative frequency
- sample space
- theoretical probability
- tree diagram
- Venn diagram

The order in which the material is presented has been changed for this edition. Experimental probability has been moved from the start of the chapter to the end, since the ideas behind finding experimental probabilities are grounded in the work done in calculating theoretical probabilities.

The Life tables section has been turned into an Activity at the end of the chapter.

Section A has been restructured to not only consider the different ways to represent the sample space of an experiment, but also to define an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, and also means the idea of an event has already been introduced when we come to calculating theoretical probabilities in Section B.

Another advantage of this approach is that it allows us to talk more generally about finding theoretical probabilities in Section B, without having to deal with 2-dimensional grids in a separate section. This is because we have already discussed how to identify an event's outcomes on a grid in Section A.

Finding probabilities from Venn diagrams has been moved forward in the chapter to Section C. This is because we only deal with single events, so it makes sense to deal with it before we move on to compound events. The method for finding unknown numbers in regions has been updated to reflect the more formal method described in Chapter 8. The only difference here is that, once the elements in the relevant regions have been found, we divide through by $n(U)$ to find the corresponding probability.

Section I from the previous edition (Sampling with and without replacement) has been absorbed into the Probabilities from tree diagrams section, since all of this work is done using tree diagrams anyway, and there is little value in placing them in separate sections.

CHAPTER 26: STATISTICS

- **A** Data collection
- **B** Types of data
- **C** Discrete numerical data
- **D** Continuous numerical data
- **E** Describing the distribution of data
- **F** Measures of centre
- **G** Measures of spread
- **H** Comparing numerical data

Keywords:

- back-to-back bar chart
- back-to-back histogram
- back-to-back stem-and-leaf plot
- biased sample
- bimodal
- bimodal distribution
- categorical data
- categorical variable
- census
- class interval
- column graph
- continuous numerical variable
- data
- discrete numerical variable
- distribution
- dot plot
- frequency histogram
- grouped histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value
- modal class
- mode
- negatively skewed distribution
- numerical data
- numerical variable
- outlier
- population
- positively skewed distribution
- range
- sample
- scale
- side-by-side column graph
- statistics
- stem-and-leaf plot
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

We begin this chapter with the Data collection section which was previously at the end of the chapter. We made this change for the same reasons as in Year 8.

The "Discrete numerical data" section has been split into three: ungrouped discrete data, grouped discrete data, and stem-and-leaf plots.

Describing the distribution of a data set is new for this year level, so we have decided to put this material in its own section. It is also now introduced *after* continuous numerical data, because we feel that it makes more sense to talk about the shape of data after we have covered all the data types. Outliers in the context of graphs are also introduced here.

The Discussion at the start of Section F deals with the use of the word "bimodal" in terms of frequency versus shape. A data set described as having a "bimodal distribution" may not necessarily have two modes. However, what we mean by "bimodal" should be clear from the context and how the question is worded.

Investigation 1 on page 459 is an opportunity for students to see how changing the sample size affects the accuracy of estimates of measures of centre.

The Comparing numerical data section is mostly unchanged from the first edition, except for the inclusion of grouped histograms. These are essentially histograms in which data for three or more groups are shown on the same axes. Unlike side-by-side column graphs, the columns for the groups are drawn overlapping, as if they are semi-transparent. The reason for this is because we cannot have gaps between the columns in a histogram. Due to their complexity, we have refrained from asking students to draw grouped histograms themselves in the exercise questions. Instead, we focus on the interpretation of them.

Investigation 2 at the end of Section H is an opportunity for students to explore a much larger data set in a spreadsheet. Rather than counting data values and doing calculations by hand, we feel that this Investigation is best done using the functions provided in the spreadsheet software used to open the data. The “Pivot table” sheet may not work in spreadsheet software other than Microsoft Excel. However, similar functionality should be available in other spreadsheet software.

CHAPTER 27: NETWORKS

A Networks
B Routes on networks
C Shortest route problems
D Eulerian networks
E Planar networks
F Euler’s formula

Keywords:

- adjacent vertices
- closed route
- connected network
- degree
- disconnected network
- edge
- Eulerian network
- Eulerian trail
- Euler’s formula
- face
- network
- node
- not traversible
- open route
- planar network
- semi-Eulerian network
- semi-Eulerian trail
- shortest route problem
- trail
- vertex
- walk
- weighted network

This online chapter has been put in the Year 9 book because, in the consultation version of the updated syllabus, much of the material for networks occurred in Year 9. We felt there was not enough content about networks in the syllabus to warrant a networks chapter in Year 9 and Year 10, so decided to combine the content into a chapter at Year 9. Since the publication of our books, the final version of the syllabus moved all of the networks content to Year 10. It is therefore advisable for students to complete this chapter, even though networks are not mentioned in the Year 9 syllabus.

One of the most challenging aspects for students starting their study of networks is the amount of terminology involved. Section A is designed to allow students to become familiar with the concept of a network, as well as the terminology used. Students should realise that a network need not refer to a physical connection between objects. For example, some networks in Exercise 27A describe friendships between people, or matches played in a tournament.

In Section B, students use terminology to describe routes between vertices. For students looking to study General Mathematics in Years 11 and 12, they will encounter more terminology about routes in those years.

In Section C, students investigate the shortest path between vertices on a weighted network. While this is effectively done by trial and error here, the Research task at the end of the section asks students to explore Dijkstra’s shortest path algorithm. Again, this algorithm will be studied further in General Mathematics in Years 11 and 12.

In the Discussion at the end of Section D, students should discover that *every* network contains an even number of vertices of odd degree. To explain this, students should first think about why the total sum of the degrees of the vertices in a network must be even. Then, they should think whether this would be possible if there were an *odd* number of vertices of odd degree.

The work on planar networks in Section E, while not explicitly a part of the syllabus, is essential for talking sensibly about Euler’s formula in Section F. Students should be encouraged to explore how Euler’s formula can be extended to polyhedrons in the Activity at the end of Section F.