Chapter 3

Percentage

Contents:
A Percentage
B The unitary method in percentage
C Finding a percentage of a quantity
D Percentage increase and decrease
E Percentage change using a multiplier
F Finding the original amount
G Simple interest
H Compound interest
Percentages are commonly used every day around us. We may see headlines like:

- Imports now taxed at 10%.
- 40% off sale this week.
- Earn 6.5% on your money.
- Opera attendances down by 8%.
- 65% of children are overweight.

An understanding of percentages and how to operate with them is therefore vital.

**OPENING PROBLEM**

Roger’s Racquets specialises in selling tennis racquets. The owner Roger purchases 120 racquets for $80 each. He applies a profit margin of 70% to the racquets, but finds he cannot sell any at that price. Consequently he has a 15% discount sale.

Consider the questions below:

- **a** What was the price of each racquet before the sale?
- **b** What was the price of each racquet after discounting?
- **c** What was the percentage profit made on the cost price of each racquet?
- **d** If 80% of the racquets were sold in the sale, how much profit was made?
- **e** What is the overall percentage return on costs if the remaining racquets are given away?

We use percentages to compare an amount with a whole which we call 100%.

% reads “per cent” which is short for *per centum*.

Loosely translated from Latin, per cent means *in every hundred.*

If an object is divided into one hundred parts then each part is called 1 per cent, written 1%.

Thus, $\frac{1}{100} = 1\%$ and $\frac{100}{100} = 100\%$

So, a percentage is like a fraction which has denominator 100.

In general:

$$\frac{x}{100} = x\%$$
CONVERTING FRACTIONS AND DECIMALS INTO PERCENTAGES

All fractions and decimals can be converted into percentages. We can do this either by:
- writing the fraction or decimal as a fraction with denominator 100, or
- multiplying by 100%, which is really just \( \frac{100}{100} \) or 1.

### Example 1

Convert each of the following to a percentage:

- \( \frac{9}{20} \)
- 3

\[
\begin{align*}
\text{a} & \quad \frac{9}{20} \\
& \quad \frac{9 \times 5}{20 \times 5} \\
& \quad \frac{45}{100} \\
& \quad 45\% \\
\text{b} & \quad 3 \\
& \quad \frac{3 \times 100}{1 \times 100} \\
& \quad 300\% \\
\end{align*}
\]

### Example 2

Convert each of the following to a percentage:

- 0.46
- 1.35

\[
\begin{align*}
\text{a} & \quad 0.46 \\
& \quad \frac{46}{100} \\
& \quad 46\% \\
\text{b} & \quad 1.35 \\
& \quad \frac{135}{100} \\
& \quad 135\% \\
\end{align*}
\]

### Example 3

Convert these into percentages by multiplying by 100%:

- \( \frac{3}{5} \)
- 0.042

\[
\begin{align*}
\text{a} & \quad \frac{3}{5} \\
& \quad \frac{3 \times 100\%}{5 \times 1} \\
& \quad 60\% \\
\text{b} & \quad 0.042 \\
& \quad 0.042 \times 100\% \\
& \quad 4.2\% \\
\end{align*}
\]

### EXERCISE 3A.1

1. Write the following as fractions with denominator 100 and hence convert to a percentage:

- \( \frac{7}{10} \)
- \( \frac{3}{4} \)
- \( \frac{7}{20} \)
- \( \frac{13}{25} \)
- 1
- 4
- \( \frac{11}{20} \)
- \( \frac{11}{10} \)
2 Write each of the following as a percentage:
   a 0.06  b 0.1  c 0.92  d 0.75
   e 1.16  f 5.27  g 0.064  h 1.024
3 Convert the following into percentages by multiplying by 100%:
   a \( \frac{2}{5} \)  b 0.82  c 0.95  d 1.085
   e 5.16  f 1.012  g 3\%\frac{1}{7} \)  h \( \frac{17}{20} \)
   i \( \frac{1}{9} \)  j 2\%\frac{5}{8} \)  k 1\%\frac{3}{5} \)  l 3\%\frac{3}{11} \)

**CONVERTING PERCENTAGES INTO FRACTIONS AND DECIMALS**

A percentage may be converted into a fraction by first writing the percentage with a denominator of 100, then expressing it in its lowest terms.

<table>
<thead>
<tr>
<th>Example 4</th>
<th>Self Tutor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express as fractions in lowest terms:</td>
<td>a 115%  b 12%\frac{1}{2}</td>
</tr>
<tr>
<td>a 115% = \frac{115}{100} = \frac{115}{100} \div 5 = \frac{23}{20}</td>
<td></td>
</tr>
<tr>
<td>b 12%\frac{1}{2} = \frac{12.5}{100} = \frac{12.5}{100} \div 125 = \frac{1}{8}</td>
<td></td>
</tr>
</tbody>
</table>

Percentages may be converted into decimals by shifting the decimal point two places to the left. This is equivalent to dividing by 100%.

<table>
<thead>
<tr>
<th>Example 5</th>
<th>Self Tutor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express as decimals:</td>
<td>a 88%  b 116%</td>
</tr>
<tr>
<td>a 88% = 088%. = 116%  b 116% = 116%.= 1.16</td>
<td></td>
</tr>
</tbody>
</table>

**EXERCISE 3A.2**

1 Express as fractions in lowest terms:
   a 85%  b 42%  c 105%  d 15%
   e 48%  f 7\%\frac{1}{2} \)  g 6\%\frac{1}{4} \)  h 132%
   i 16\%\frac{2}{3} \)  j 33\%\frac{1}{3} \)  k 160%  l 0.25%
Express as decimals:

a 92%  
\[0.92\]

b 106%  
\[1.06\]

c 112.4%  
\[1.124\]

d 88.2%  
\[0.882\]

e 7.5%  
\[0.075\]

f 1%  
\[0.01\]

g 256%  
\[2.56\]

h 0.05%  
\[0.0005\]

i 1150%  
\[11.5\]

j 0.0037%  
\[0.000037\]

k 342%  
\[3.42\]

l 63.7%  
\[0.637\]

**ONE QUANTITY AS A PERCENTAGE OF ANOTHER**

We can compare quantities using percentages. To find one quantity as a percentage of another, we write the first as a fraction of the second, then multiply by 100%.

**Example 6**

Express as a percentage:

a  Mike ran 10 km out of 50 km

\[\frac{10}{50} \times 100\% = 20\%\]  
So, Mike ran 20% of 50 km.

b  Rani spent 5 months of the last two years overseas.

\[\frac{5}{24} \times 100\% \approx 20.8\%\]  
So, Rani spent about 20.8% of the last 2 years overseas.

3  Express as a percentage:

a  40 marks out of 50 marks  
b  21 marks out of 35 marks  
c  5 km out of 40 km  
d  500 m out of 1.5 km  
e  8 km out of 58 km  
f  130 kg out of 2.6 tonnes  
g  4 hours out of 1 day  
h  3 months out of 3 years

4  Anastasia was given €20 pocket money and Emma was given €24. Anastasia saved €7 while Emma saved €9. Who saved the greater percentage of their pocket money?

5  Matt spent $40 on jeans, $25 on a top and $65 on shoes. He received $20 change from $150. What percentage of his money did Matt spend on:

a  jeans  
b  a top  
c  shoes  
d  all of his clothes?

6  Maya scored 32 out of 40 for a Maths test and 41 out of 55 for a Science test. For which test did she score a lower percentage?
INVESTIGATION 1  SPORTING INJURIES

The graphs below show the number of players involved in eight different sports in England and the number of injuries suffered by players involved in those same eight sports.

What to do:

Investigate, using your calculator:

1 the total number of injuries per year in all 8 sports

2 injuries per year for each of the eight sports, expressed as a percentage of total injuries per year

3 the total number of people in England playing these 8 sports

4 the percentage of the total number of players playing each sport

5 the injury rate for each sport using

\[
\text{Injury rate} = \frac{\text{number of injuries per year}}{\text{number playing that sport}} \times 100\%.
\]

6 Use the injury rates to decide which sport appears to be the:
   a most dangerous
   b safest.

THE UNITARY METHOD IN PERCENTAGE

Sometimes we know a certain percentage of the whole amount. For example,

Maddie knows that 16% of her wage is deducted for tax. Her payslip shows that $120 is taken out for tax. She wants to know her total income before tax.

The unitary method can be used to solve such problems.
There is no need to actually calculate \( \frac{120}{16} \): The final answer can be found in one step by multiplying \( \frac{120}{16} \) by 100.

Example 7

Find 100% of a sum of money if 16% is $120.

16% of the amount is $120
\[ \therefore 1\% \text{ of the amount is } \frac{120}{16} = 7.50 \]
\[ \therefore 100\% \text{ of the amount is } 7.50 \times 100 = 750. \]

Example 8

Find 60% of a sum of money if 14% is $728.

14% is $728
\[ \therefore 1\% \text{ is } \frac{728}{14} = 52 \]
\[ \therefore 60\% \text{ is } 52 \times 60 = 3120. \]

Example 9

82% of fans at a basketball match support the Lakers. If there are 24,026 Lakers fans at the match, how many people attend the match?

82% is 24,026 fans
\[ \therefore 1\% \text{ is } \frac{24026}{82} = 293 \text{ fans} \]
\[ \therefore 100\% \text{ is } 293 \times 100 = 29300 \]

So, 29,300 fans attend the match.

EXERCISE 3B

1 Find 100% if:
   a 20% is $360
   b 24% is 192 kg
   c 9% is 225 mL
   d 15% is 450 kg
   e 87% is $1044
   f 95% is 342 mL
   g 12% is 66 L
   h 35% is 252 kg
   i 47% is $585.

2 Find:
   a 30% if 7% is $126
   b 72% if 11% is 176 kg
   c 5% if 48% is $816
   d 15% if 90% is 1890 mL
   e 95% if 6% is 55 kg
   f 4% if 85% is $1000.
3. 24% of the passengers on board a cruise ship are children. If there are 210 children aboard, determine the total number of passengers on board the ship.

4. 80% of a plumbing contractor’s income was from government contracts. If his income for the year from government contracts was $74,000, find his total annual income.

5. A country town has 1536 female residents. 48% of its population is female. Find the town’s total population.

6. In the local high school 18% of the students play football and 32% play netball. If 126 students play football, how many students:
   a. attend the school
   b. play netball?

C

**FINDING A PERCENTAGE OF A QUANTITY**

To find a percentage of a quantity, convert the percentage to a fraction or a decimal and then multiply.

### Example 10

<table>
<thead>
<tr>
<th>Find:</th>
<th>a</th>
<th>15% of 400 kg</th>
<th>b</th>
<th>4.5% of €210</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>15% of 400 kg</td>
<td>b</td>
<td>4.5% of €210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 0.15 x 400 kg</td>
<td>= 0.045 x 210</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 60 kg</td>
<td>= €9.45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Example 11

Sandra scored 86% in her exam out of 150. What mark did she score?

- Sandra scored 86% of 150
- = 0.86 x 150
- = 129
- So, Sandra scored 129 marks.

### EXERCISE 3C

1. Find:
   a. 30% of 90 kg
   b. 25% of €170
   c. 4% of 50 L
   d. 75% of 40 km
   e. 6.5% of $540
   f. 95% of 5 m
   g. 47 1/2% of £1400
   h. 1 1/2% of $53,600
2 Solve the following problems:
   a Su-la scored 45% in her test out of 80. What mark did she score?
   b John scored 72% for an examination marked out of 150. How many marks did he actually score out of 150?
   c A mixture of petrol and oil for a two-stroke lawn mower contains 85% petrol. How much oil is required for 18 litres of the fuel mixture?
   d A real estate agent receives $4\frac{1}{2}$% commission on the sale of all property she handles. How much does she receive for a house she sells for £148,500?
   e A share farmer receives 65% of the proceeds of the sale of a crop of wheat. If the wheat is sold for $62,400, how much does he receive?
   f To insure goods to send them overseas it costs the exporter $2\frac{1}{7}$% of the value of the goods. If the goods are valued at €16,400, what will the insurance cost?

3 38.8% of Canada’s population live in Ontario. The population of Ontario is 12.9 million.
   a Use the unitary method to find the population of Canada.
   b If 2.8% of Canadians live in Nova Scotia, how many actually live in Nova Scotia?

**PERCENTAGE INCREASE AND DECREASE**

Every day we are faced with problems involving money. Many of these situations involve percentages. For example, we use percentages to describe profit, loss and discount.

Profit is an example of **percentage increase**. Loss and discount are examples of **percentage decrease**.

**PROFIT AND LOSS**

Profit or loss is the difference between the selling price and the cost price of an item.

Profit or loss = selling price − cost price.

A profit occurs if the selling price is higher than the cost price.

A loss occurs if the selling price is lower than the cost price.

Businesses make a profit by buying goods cheaply and then marking up or increasing the price when they sell them.
A TV set is purchased for $450 and is marked up by 30%.

Find:

(a) the profit
(b) the selling price.

\[ \text{Profit} = 30\% \text{ of cost price} \]
\[ = 30\% \times 450 \]
\[ = 0.30 \times 450 \]
\[ = 135 \]

\[ \text{Selling price} = \text{cost price} + \text{profit} \]
\[ = 450 + 135 \]
\[ = 585 \]

A retailer will often express their profit or loss as a percentage of the cost price.

For a profit we find the \textit{percentage increase} in the price.

For a loss we find the \textit{percentage decrease} in price.

A bicycle costs $240 and is sold for $290. Calculate the profit as a percentage of the cost price.

\[ \text{Profit} = \text{selling price} - \text{cost price} \]
\[ = 290 - 240 \]
\[ = 50 \]

\[ \text{profit as a percentage of cost price} \]
\[ = \frac{\text{profit}}{\text{cost price}} \times 100\% \]
\[ = \frac{50}{240} \times 100\% \]
\[ \approx 20.8\% \]

EXERCISE 3D.1

1. For the following items, find the: i) profit or loss  ii) selling price
   a) a bicycle is purchased for $300 and marked up 80%
   b) a ring is purchased for €650 and marked down 45%
   c) a house is purchased for £137 000 and sold at a 15% profit
   d) a car is purchased for ¥2 570 000 and sold at a 22% loss.

2. A bicycle costs $260 and is sold for $480. Calculate the profit as a percentage of the cost price.

3. A greengrocer buys fruit and vegetables from the market and sells them at a 25% mark up. On one particular morning, her fruit and vegetables cost her €500. If she sells all of her produce, find: a) her profit b) her total income.
4 A 30 m roll of wire mesh was bought wholesale for £216. If it is sold for £8.50 per metre, find the profit and express it as a percentage of the wholesale or cost price.

Example 14

Ali bought shares in Boral at $10.50 per share, but was forced to sell them at $9.30 each. Calculate:

a her loss per share       b the loss per share as a percentage of the cost.

<table>
<thead>
<tr>
<th>a</th>
<th>Loss</th>
<th>b</th>
<th>% loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= selling price – cost price</td>
<td>= loss / cost × 100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= $9.30 – $10.50</td>
<td>= $1.20 / $10.50 × 100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= –$1.20</td>
<td>≈ 11.4%</td>
<td></td>
</tr>
<tr>
<td>i.e.,</td>
<td>a $1.20 per share loss.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 A used car firm pays $6000 for a car, but, because of financial difficulties, has to sell it immediately and receives only $4920 for the sale. Find the loss incurred by the used car firm and express this loss as a percentage of the cost price.

6 Ulrich and Jade purchased a new house for £320 000. Due to interest rate rises after 3 years they were unable to afford their mortgage repayments and had to sell the house for £285 000. Find:

a the loss incurred        b the loss as a percentage of their total costs.

7 A hardware store has a closing down sale. They advertise an aluminium ladder at $256. If the wholesale or cost price of the ladder was $274, find the loss and express it as a percentage of the cost price.

DISCOUNT

A discount is a reduction in the marked price of an item.

When retail stores advertise a sale, they offer a percentage off the marked price of most goods.

Discounts are often given to tradespeople as encouragement to buy goods at a particular store.
A store offers a discount of 15% off the marked price of a shirt selling for $49.
Find the:  

\( a \)  
Discount = \( 15\% \) of $49  
\( = 0.15 \times 49 \)  
\( = 7.35 \)

\( b \)  
Sale price = marked price – discount  
\( = 49 - 7.35 \)  
\( = 41.65 \)

**EXERCISE 3D.2**

1 Find the discount offered on the following items and hence find the sale price:
   \( a \) a pair of shoes marked at £70 and discounted 40%  
   \( b \) a suit marked at £150 and discounted 25%  
   \( c \) a cap marked at $24 and discounted 12\(\frac{1}{2}\)%.

2 A plumber buys supplies worth €220 but is given a 5% discount. What does she save with the discount?

3 A builder buys timber worth £4800 but is given a 12% discount. What does he pay for the timber?

4 A dressmaker buys material in bulk. It is marked at ¥13200 but she is given a 7\(\frac{1}{2}\)% discount. How much does she actually pay for the material?

Kylie buys a pair of jeans marked at $90 but only pays $76.50. What percentage discount was she given?

\[
\text{Discount} = 90 - 76.50 = 13.50
\]
\[
\therefore \text{\% discount} = \frac{\text{discount}}{\text{marked price}} \times 100\% = \frac{13.50}{90} \times 100\% = 15\% 
\]

So, Kylie was given 15% discount.

5 Ronan purchases a CD marked at €28 but actually pays €23.80. What percentage discount was he given?

6 Nghia saw a car advertised for sale at $17,875, having been discounted from $27,500. Calculate the percentage discount.

7 A supermarket employee buys groceries worth ¥7600 but is only charged ¥7030. What employee discount did she receive?
In the exercise on profit and loss, we dealt with percentage change. Another simple method for working with percentage change is to use a multiplier.

For example:

- If we increase an amount by 30%, we will have \(100\% + 30\% = 130\%\) of the amount. So, to increase an amount by 30% we multiply by \(\frac{130}{100}\) or 1.3. 1.3 is the multiplier.

- If we decrease an amount by 30%, we will have \(100\% - 30\% = 70\%\) of the amount. So, to decrease an amount by 30% we multiply by \(\frac{70}{100}\) or 0.7. 0.7 is the multiplier.

### Example 17

What multiplier corresponds to:

a. a 50% increase  
b. a 12% decrease?

### Example 18

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Increase $300 by 20%.</td>
</tr>
<tr>
<td>b</td>
<td>Decrease $300 by 20%.</td>
</tr>
</tbody>
</table>
| a | New amount  
   | = 120% of $300  
   | = 1.2 \times 300  
   | = $360 |
| b | New amount  
   | = 80% of $300  
   | = 0.8 \times 300  
   | = $240 |

### Exercise 3E.1

1. What multiplier corresponds to:

   a. 40% increase  
   b. 6% increase  
   c. 20% decrease  
   d. 15% decrease  
   e. 42% decrease  
   f. 12% increase?

2. Perform the following calculations:

   a. Increase $120 by 15%.  
   b. Increase 450 kg by 20%.  
   c. Decrease £4800 by 24%.  
   d. Decrease $720 by 8%.  
   e. Increase 5000 hectares by 120%.  
   f. Decrease 1600 tonnes by 12%.  
   g. Decrease 12,500 m² by 1.46%.  
   h. Increase €125,672 by 0.13%.
FINDING A PERCENTAGE CHANGE

The multiplier method can be used to find the percentage increase or decrease given the original and new amounts. We do this by expressing the new amount as a fraction of the original amount and then converting the result to a percentage.

\[
\text{multiplier} = \frac{\text{new amount}}{\text{original amount}}
\]

Finding the fraction \(\frac{\text{new amount}}{\text{original value}}\) can always be used to find percentage changes.

Example 19

Find the percentage increase when $160 changes to $180.

\[
\begin{align*}
\text{new amount} & \quad \text{original value} \\
$180 & \quad $160 \\
= 1.25 & \quad \{\text{the multiplier is 1.25}\} \\
= 1.25 \times 100\% & \quad \{\text{decimal to percentage}\} \\
= 125\% & \quad \{100\% + 12.5\%\}
\end{align*}
\]

So, there is a 12.5% increase.

Exercise 3E.2

1. Find the percentage increase in the following, to 1 decimal place if necessary:
   a. £80 changes to £96
   b. €14 000 changes to €16 000
   c. 32 hours changes to 37.5 hours
   d. 180 cm changes to 185 cm
   e. 42 kg changes to 49 kg
   f. $156 000 changes to $164 000
   g. 3.5 kg changes to 7 kg
   h. 52.4 L changes to 61.7 L

2. My dairy herd produced a daily average of 467 L of milk last year. This year production has increased to 523 L. What is the percentage increase in milk production?

Example 20

Find the percentage decrease when 80 kg is reduced to 72 kg.

\[
\begin{align*}
\text{new amount} & \quad \text{original value} \\
72 \text{ kg} & \quad 80 \text{ kg} \\
= 0.9 & \quad \{\text{the multiplier is 0.9}\} \\
= 0.9 \times 100\% & \quad \{\text{decimal to percentage}\} \\
= 90\% & \quad \{100\% - 10\%\}
\end{align*}
\]

So, we have a 10% decrease.
3 Find the percentage decrease in the following:
   a $80 to $70  
   b 95 kg to 90 kg  
   c 60 hours to 40 hours  
   d 8 km to 4 km  
   e $155 to $140  
   f €16 to €4  

4 Increase $1000 by 10% and then decrease your answer by 10%. What do you notice?  

5 My parents increased my pocket money by 10% and then three months later increased it by a further 10%. My father said this was an increase of 21%. Can you explain this?  

APPLICATIONS OF THE MULTIPLIER  

A simple application of the multiplier is in business problems where we are calculating the selling price. We are actually increasing or decreasing the cost price and so the multiplier can be used.  

\[
\text{Selling price} = \text{cost price} \times \text{multiplier}
\]

Example 21  

A warehouse owner buys a refrigerator for $750 and marks it up by 35%. At what price does the owner sell the refrigerator?  

\[
100\% + 35\% = 135\% \\
\therefore \text{multiplier} = 1.35 \\
\text{selling price} = \text{cost price} \times \text{multiplier} \\
= 750 \times 1.35 \\
= 1012.50 \\
\]

The refrigerator is sold for $1012.50.  

EXERCISE 3E.3  

1 When a car priced at €14200 is bought, a further 10% must be added for tax. What is the selling price of the car?  

2 A leather coat costs a fashion store $150. They will sell it for a 70% profit. Find:
   a the selling price of the coat  
   b the profit as a percentage of the selling price.  

3 A real estate company buys a block of units for €326000. They spend €22000 on renovations and repairs. Three months later they are able to sell the units at a profit of 11% on their total investment. Find the total sale price for the block of units.
INVESTIGATION 2 DOUBLING AN INVESTMENT

Trevor invests in $1000 worth of shares. He expects the value of his investment to increase by 10% each year.

Trevor decides that he will sell the shares when they have doubled in value.

The purpose of this investigation is to find how long it takes for any investment to double in value at a particular rate. Doubling will usually occur during a year, but we are only interested in the whole number of years immediately after the doubling has occurred.

What to do:

1. Consider doubling the value of a $1000 investment which is increasing by 10% p.a. each year.

   For an increase of 10%, we must multiply our investment amount by 110% = 1.1. So, our multiplier for each year is 1.1.
### New amount

$1000 \times 1.1 = $1100 \text{ at the end of the first year.}

Copy and complete the given table:

<table>
<thead>
<tr>
<th>Number of years</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1000</td>
</tr>
<tr>
<td>1</td>
<td>$1100</td>
</tr>
<tr>
<td>2</td>
<td>$1210</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
</tbody>
</table>

How long will it take for the investment to double in value?

2 Would the answer be different if the initial value of $1000 was changed? Try some other initial values to see what happens.

3 Investigate what happens with other rates of increase, such as 4%, 6%, 8%, and 12%. You could use the spreadsheet which follows by clicking on the icon. 

**Hint:** For 4%, enter 0.04 in cell C1.

4 Graph your results, with investment rates on the horizontal axis and doubling time on the vertical axis. Comment on your results.

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### FINDING THE ORIGINAL AMOUNT

It is often useful to know what the original value of an item was before a percentage increase or decrease occurred.

For example, suppose an item is marked up by 30% and its new price is £156. How can we find its original price?

The following example illustrates a method for doing this.

**Example 23**

The price of a TV set is marked up by 25% for sale. Its selling price is $550. For what price did the shopkeeper buy the TV set?

\[
\text{cost price} \times \text{multiplier} = \text{selling price} \\
\therefore \text{cost price} \times 1.25 = 550 \quad \{100\% + 25\% = 125\% = 1.25\} \\
\therefore \text{cost price} = \frac{550}{1.25} = 440
\]

So, the television set cost the shopkeeper $440.
EXERCISE 3F

1 Find the original amount given that:
   a after an increase of 25% the price was RM250
   b after an increase of 35% the price was $243
   c after a decrease of 10% the price was £81
   d after a decrease of 17% the price was €37.35
   e after a decrease of 37.5% the price was 115 pesos
   f after a decrease of 22 1/2% the price was €9300

2 ‘Blacks Furniture Mart’ sells a lounge suite for $3280.50, making a profit of 35% on the cost price. How much did the business pay for the lounge suite?

3 A retailer sells a microwave oven for €640. This is a 25% profit on the cost price. How much did the retailer pay for the microwave oven?

4 An electrical firm sells a washing machine for $383.50, making a 30% profit on the wholesale or cost price. Find the wholesale price of the machine.

5 Jason sells a bicycle for $247 at a loss of 35%. What did Jason pay for the bicycle originally?

G SIMPLE INTEREST

When a person borrows money from a lending institution such as a bank or a finance company, the borrower must repay the loan in full, and also pay an additional interest payment. This is a charge for using the institution’s money.

Similarly, when money is invested in a bank, the bank pays interest on any deposits.

SIMPLE INTEREST

If the interest is calculated each year as a fixed percentage on the original amount of money borrowed or invested, then the interest is called simple interest.

For example, suppose $8000 is invested for 5 years at 10% per annum or per year simple interest.

The simple interest paid for 1 year = 10% of $8000
   = 0.1 × $8000
   = $800

Thus, the simple interest for 2 years = 10% of $8000 × 2
   = 0.1 × $8000 × 2
   = $1600

Thus, the simple interest for 5 years = 10% of $8000 × 5
   = 0.1 × $8000 × 5
   = $4000
These observations lead to the simple interest formula.

**SIMPLE INTEREST FORMULA**

If $C$ is borrowed or invested for $n$ years at $r\%$ p.a. (per annum) simple interest, the simple interest $I$ can be calculated using the formula:

$$I = Crn$$

where $C$ is the **principal**, $r$ is the **flat rate of interest per annum**, and $n$ is the **time or duration** of the loan in **years**.

### Example 24

Find the simple interest payable on an investment of $20\,000 at 12\%$ p.a. over a period of 4 years.

- $C = 20\,000$
- $r = 12 \div 100 = 0.12$
- $n = 4$

Now $I = Crn$

\[ \therefore I = 20\,000 \times 0.12 \times 4 \]

\[ \therefore I = 9600 \]

\[ \therefore \text{simple interest is } $9600.\]

### Example 25

Calculate the simple interest payable on an investment of $15\,000 at 8\%$ p.a. over 9 months.

- $C = 15\,000$
- $r = 8 \div 100 = 0.08$
- $n = \frac{9}{12} = 0.75$

Now $I = Crn$

\[ \therefore I = 15\,000 \times 0.08 \times 0.75 \]

\[ \therefore I = 900 \]

\[ \therefore \text{simple interest is } $900.\]

In some areas of finance, sums of money may be invested over a period of **days**. However, the interest rate is still normally quoted per annum, so the time period $n$ in the formula must be in **years**. So, the number of days **must be divided by 365**.

### Example 26

Determine the simple interest payable on an investment of $100\,000 at 15\%$ p.a. from April 28th to July 4th.

From April 28th there are 2 days left in April.

\[ \therefore \text{2 days left in April} \quad \text{we exclude the first day (April 28)} \]

31 days in May

30 days in June

4 days in July \quad \text{we include the last day (July 4)}

67 days
\[ C = 100\,000 \quad r = 15 \div 100 = 0.15 \quad n = \frac{67}{365} \approx 0.183\,562 \]
\[ \therefore I = 100\,000 \times 0.15 \times 0.183\,562 \]
\[ \therefore I = 2753.42 \]

So, the interest payable is $2753.42

**EXERCISE 3G**

1. Find the simple interest payable on an investment of:
   a) $4000 at 8% p.a. for 5 years
   b) £1500 at 11% p.a. for 3 years
   c) €2500 at 101\% p.a. for 2 years
   d) $20,000 at 121\% p.a. for 4 years.

2. Find the simple interest payable on an investment of:
   a) $5000 at 7% p.a. over 6 months
   b) €8000 at 9% over 3 months
   c) ¥1600000 at 31\% p.a. over 10 months
   d) £11500 at 51\% p.a. over 18 months.

3. Stella Ho deposits €46,000 in a special investment account on March 17th. If the account pays 9\% p.a. simple interest and she withdraws the money on June 30th, how much will her investment have earned during this time?

4. Tony Giacomin deposited $1600 on July 3rd in a special investment account which earns 13% p.a. simple interest. On August 17th he deposited another $5600 in the account. If he closed the account on November 12th by withdrawing the total balance, calculate how much his investment has earned over this period of time.

**Example 27**

Calculate the total amount to be repaid if $5000 is borrowed for 3 years at 14% p.a. simple interest.

\[ C = 5000 \quad r = 14 \div 100 = 0.14 \quad n = 3 \]
\[ \therefore I = 5000 \times 0.14 \times 3 \]
\[ \therefore I = 2100 \]

The total repayment = principal + interest
\[ = 5000 + 2100 \]
\[ = 7100 \]

5. If £2000 is borrowed under simple interest terms, how much must be repaid after:
   a) 3 years at 5% p.a.
   b) 8 months at 12% p.a.
   c) 4 years at 81\% p.a.?

6. Jamil borrows $5400 from the finance company to buy his first car. The rate of simple interest is 13\% per annum and he borrows the money over a 5 year period. Find:
   a) the amount Jamil must repay the finance company
   b) his equal monthly repayments. **Hint:** There are 60 months in 5 years.
7 An electric guitar with all attachments is advertised at €2400. If Klaus pays a deposit of €600, he then has to borrow the remainder or balance at 12% p.a. simple interest over 3 years. What are his monthly repayments?

**COMPOUND INTEREST**

When you deposit money in the bank, you are in effect, **lending** the money to the bank. The bank in turn uses your money to lend to other people. The bank will pay you interest to encourage your custom, and they charge interest to borrowers at a higher rate. This way the bank makes a profit.

If you leave the money in the bank for a period of time, the interest is automatically added to your account.

After the interest is added to your account, it will also earn interest in the next time period.

Consider the following example:

$1000 is placed in an account earning interest at a rate of 10% p.a. The interest is allowed to **compound** itself for three years. We say it is earning 10% p.a. **compound interest**.

We can show this in a table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount at beginning of year</th>
<th>Compound Interest</th>
<th>Amount at end of year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000</td>
<td>10% of $1000 = $100</td>
<td>$1100</td>
</tr>
<tr>
<td>2</td>
<td>$1100</td>
<td>10% of $1100 = $110</td>
<td>$1210</td>
</tr>
<tr>
<td>3</td>
<td>$1210</td>
<td>10% of $1210 = $121</td>
<td>$1331</td>
</tr>
</tbody>
</table>

After 3 years there is a total of $1331 in the account. We have earned $331 in compound interest.

If we construct a similar table for $1000 in an account earning 10% p.a. simple interest for 3 years, we can compare the values of the 2 different types of interest.

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount at beginning of year</th>
<th>Simple Interest</th>
<th>Amount at end of year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000</td>
<td>10% of $1000 = $100</td>
<td>$1100</td>
</tr>
<tr>
<td>2</td>
<td>$1100</td>
<td>10% of $1100 = $110</td>
<td>$1200</td>
</tr>
<tr>
<td>3</td>
<td>$1200</td>
<td>10% of $1200 = $120</td>
<td>$1300</td>
</tr>
</tbody>
</table>

After 3 years there is a total of $1300 in the account, so we have earned $300 in simple interest.

Comparing the two,
Notice that the principal on which the interest is calculated is different for the two forms of interest:

- For simple interest we always use the initial amount invested as the principal in each calculation.
- For compound interest we use the amount at the end of the previous year as the principal in each calculation.

For the same rate of interest, therefore, we will always earn more interest from compound interest accounts than from simple interest accounts over the same length of time.

### Example 28

How much interest will I earn if I invest $10,000 for 3 years at:

- a 15% p.a. simple interest
- b 15% p.a. compound interest?

#### a
We use the simple interest formula where \( C = 10,000 \), \( r = 0.15 \), \( n = 3 \).

Now \( I = Crn \)

\[ I = 10,000 \times 0.15 \times 3 = 4500 \]

Thus, the interest is $4500.

#### b

<table>
<thead>
<tr>
<th>Year</th>
<th>Initial Amount</th>
<th>Interest</th>
<th>Final Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
<td>15% of $10,000 = $1500.00</td>
<td>$11,500.00</td>
</tr>
<tr>
<td>2</td>
<td>$11,500</td>
<td>15% of $11,500 = $1725.00</td>
<td>$13,225.00</td>
</tr>
<tr>
<td>3</td>
<td>$13,225</td>
<td>15% of $13,225 = $1983.75</td>
<td>$15,208.75</td>
</tr>
</tbody>
</table>

Interest = final amount – initial amount

\[ = $15,208.75 – $10,000 \]

\[ = $5208.75 \]

### EXERCISE 3H.1

1. Calculate:
   - a the simple interest earned on €2000 at 5% p.a. for 3 years
   - b using a table, the compound interest earned on €2000 at 5% p.a. for 3 years.

2. If £50,000 is invested at 9% p.a. compound interest, use a table to find:
   - a the final amount after 2 years
   - b how much interest was earned in the 2 year period.

3. Use a table to determine the interest earned for the following investments:
   - a €4000 at 8% p.a. compound interest for 2 years
   - b $12,000 at 6% p.a. compound interest for 3 years
   - c £500 at 3% p.a. compound interest for 3 years.
INVESTIGATION 3  COMPOUND INTEREST SPREADSHEET

A spreadsheet is an ideal way to investigate compound interest investments because it allows us to construct the table very rapidly.

Suppose $5000 is invested at 4% p.a. compound interest for 10 years.

What to do:

1. Open a new spreadsheet and enter the following:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Investment</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Interest rate</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Year</td>
<td>Initial amount</td>
<td>Interest</td>
</tr>
<tr>
<td>4</td>
<td>=A5+1</td>
<td>=B5</td>
<td>=B5*C2</td>
</tr>
</tbody>
</table>

2. Highlight the formulae in row 6. Fill down to row 14 for the 10th year of investment.

3. Format all amounts in dollars.

4. Use your spreadsheet to answer the following questions:
   a. What interest was paid in i Year 1 ii Year 10?
   b. How much is in the account after i 5 years ii 10 years?

5. Suppose $15 000 is invested at 6% p.a. compound interest for 10 years. Enter 15 000 in B1 and 0.06 in B2. For this investment, answer a and b in question 3 above.

6. Suppose $8000 is invested at 5% p.a. compound interest to double in value? Hint: Enter 8000 in B1, 0.05 in B2, and fill down further.

6. What compound interest rate is needed for $12 000 to double in value after 6 years? Hint: Enter 12 000 in B1 and repeatedly change the interest rate in B2.

THE COMPOUND INTEREST FORMULA

Suppose you invest $1000 in the bank for 3 years, earning 10% p.a. compound interest.

Since the interest rate is 10% p.a., your investment increases in value by 10% each year. Its new value is $1000 \times 1.1 = 1100$ of the value at the start of the year, which corresponds to a multiplier of 1.1.

After one year your investment is worth $1000 \times 1.1 = 1100$
After two years it is worth $1100 \times 1.1 = 1210$
After three years it is worth $1210 \times 1.1 = 1331$

This suggests that if the money was left in your account for n years, it would amount to $1000 \times (1.1)^n$. 
We can write a formula for **Compound Growth**:

\[
F_v = P_v(1 + i)^n
\]

where

- \(F_v\) is the **future value**
- \(P_v\) is the **present value** or original amount
- \(i\) is the **annual interest rate** as a decimal
- \((1 + i)\) is the **multiplier**
- \(n\) is the **number of years of investment**

Notice that the formula for \(F_v\) above gives the total future value, which is the original amount plus interest.

To find the **interest only** we use:

\[
\text{Compound interest} = F_v - P_v
\]

### Example 29

**a** What will $5000 invested at 8% p.a. compound interest amount to after 2 years?

**b** How much interest is earned?

**a** An interest rate of 8% indicates that \(i = 0.08\).

For 2 years, \(n = 2\) and so

\[
F_v = P_v(1 + i)^n = $5000 \times (1.08)^2 = $5832
\]

**b** Interest earned = $5832 - $5000 = $832.

### EXERCISE 3H.2

1. **a** What will an investment of $3000 at 10% p.a. compound interest amount to after 3 years?

   **b** What part of this is interest?

2. How much compound interest is earned by investing €20 000 for 4 years at 12% p.a.?

3. £5000 is invested for 2 years at 10% p.a. What will this investment amount to if the interest is calculated as:
   - **a** simple interest
   - **b** compound interest?

4. **a** What will an investment of $30 000 at 10% p.a. compound interest amount to after 4 years?

   **b** What part of this is interest?

5. How much compound interest is earned by investing €80 000 at 9% p.a. over a 3 year period?

6. £6000 is invested for 2 years at 15% p.a. What will this investment amount to if the interest is calculated as:
   - **a** simple interest
   - **b** compound interest?
7 You have €8000 to invest for 3 years and there are 2 possible options you have been offered:

   Option 1: Invest at 9% p.a. simple interest.
   Option 2: Invest at 8% p.a. compound interest.

   a Calculate the amount accumulated at the end of the 3 years for both options and decide which option to take.
   b Would you change your decision if you were investing for 5 years?

8 What percentage increase will occur if I invest any amount over a 4 year period at 10% p.a. compound interest? **Hint:** Let the principal be 1000 of your local currency.

9 An investment of $5000 at 7% interest compounded annually over \(x\) years will grow to \(5000 \times (1.07)^x\). Enter the function \(Y_1 = 5000 \times (1.07)^x\) into a graphics calculator and use the calculator to find:

   a the value of the investment after i 5 years ii 10 years iii 20 years
   b how long it takes for the investment to increase to:
     i $10000 ii $20000 iii $40000.

Comment on your answers.

**INVESTIGATION 4**

THE RATE OF INCREASE SPREADSHEET

Click on the icon to obtain a printable investigation on finding the annual average rate of increase in an investment.

**REVIEW SET 3A**

1 What multiplier corresponds to: a an 8% increase b a 7% decrease?

2 Find the percentage change when €108 is increased to $144.

3 a Decrease $160 by 18% using a multiplier.
   b Increase 120 kg by 10% using a multiplier.

4 If 28% of a shipment of books weighs 560 kg, find the total weight of the shipment.

5 Jodie sold a dress for $224, making a loss of 30%. How much did the dress cost her?

6 Herb sold his house for £213 600 and made a 78% profit. How much did the house originally cost him?
7 See Kek inherited \(33\frac{1}{3}\%\) of her uncle’s estate of $420,000. However, she needs to pay 15% of her inheritance in tax.
   a) What percentage did she actually inherit after the tax is paid?
   b) How much did she actually inherit?

8 The local sports store buys sports shirts for $16 and adds a 40% mark up. In an end of season sale, a 25% discount is offered. Find the customer’s price for a sports shirt.

9 Determine the simple interest on a loan of €7500 for 4 years at 8% p.a.

10 Determine the compound interest earned on £50,000 at 4% p.a. over a four year period.

11 A local manufacturing business has an increase of 6% in sales. Find the original weekly sales if the business now makes sales of $8533 per week.

---

**REVIEW SET 3B**

1 What multiplier corresponds to:
   a) a 2\(\frac{1}{2}\)% decrease
   b) a 7.3% increase?

2 Increase $240 by 24% using a multiplier.

3 A digital TV marked at $3000 is discounted by 12%. Find:
   a) the discount given
   b) the selling price.

4 A television was bought for $560 and sold for $665. Find the profit as a percentage of the cost price.

5 The deposit of 40% for a concreting job costs $2400. How much will the remaining 60% cost?

6 Imran bought a cricket bat and then sold it for $250 at a profit of 25%. How much did the bat cost him?

7 Sergio exercised regularly to decrease his body weight by 14%. He now weighs 81.7 kg. How heavy was Sergio before he commenced the exercise program?

8 In the first year of business, Jennifer made a profit of €83,000. In the second year her profit increased to €98,000.
   a) By what percentage did her profit increase?
   b) What is her estimated profit for the next year assuming the same percentage increase as before?

9 a) Determine the simple interest on a loan of $7800 for 3 years at 11% p.a.
   b) Find the equal monthly repayments required to pay off the loan.

10 Determine the compound interest earned on $30,000 at 5% p.a. over a three year period.

11 Over a period of time, the value of a house increased by 15% to $455,400. Find the original value of the house.